Comment expliquer les preuves à divulgation nulle de connaissance à vos enfants ?

Pascal Lafourcade

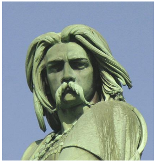
Chaire sur la Confiance Numérique

28 Mai 2014

Idea of Zero Knowledge Proof

Prover (P)

(P) convinces (V) that it knows something without revealing any information



Verifier (V)

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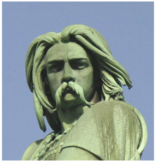

Verifier (V)

Applications:

- Authentication systems: prove its identity to someone using a password without reavealing anything about the secret.
- Prove that a praticipant behavior is correct according to the protocol (e.g. integrity of ballots in vote).
- Group signature, secure multiparty computation, e-cash ...

Outline

Motivation

Outline

Outline

Outline

Outline

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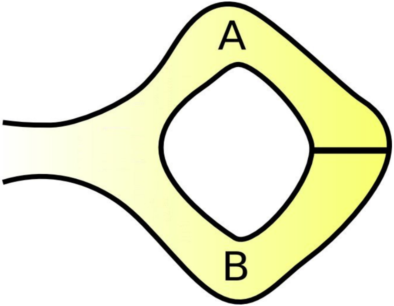
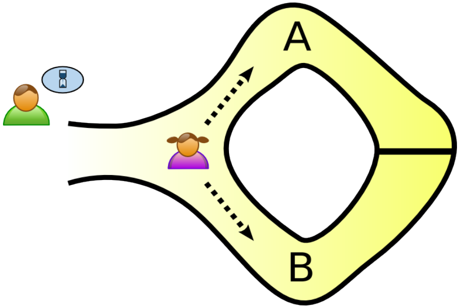
Cave example (0)



Door with a secret code

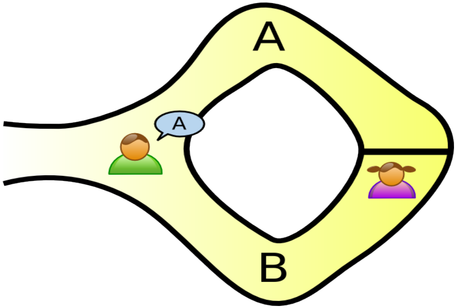
Cave example (I)

V waits outside while P chooses a path



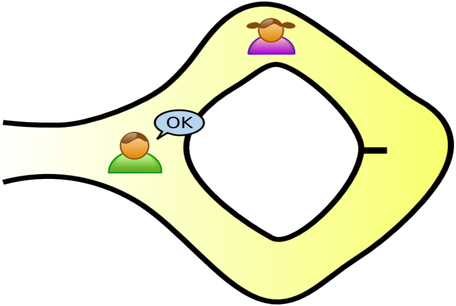
Cave example (II)

V enters and shouts the name of a path



Cave example (III)

P returns along the desired path (using the secret if necessary)



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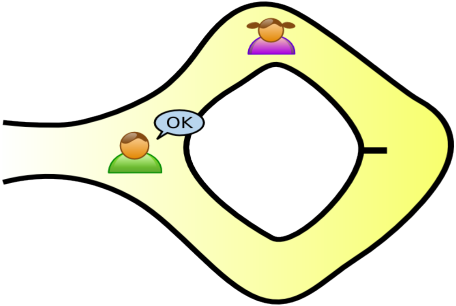

$A =$ "P does not know the secret" is equivalent to say "P is lucky"

$$Pr[A] = \frac{1}{2}$$

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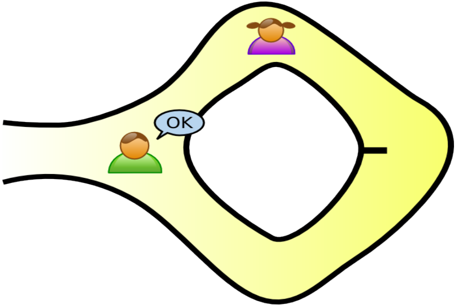

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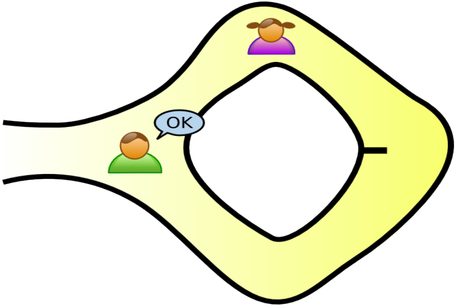

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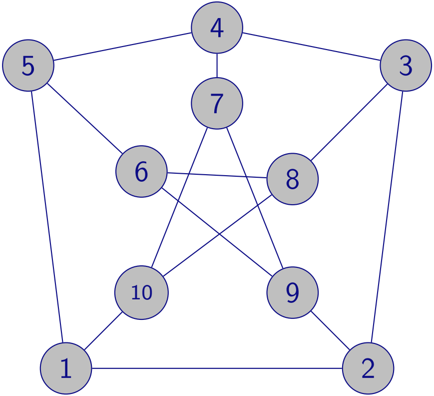
$\overline{A} =$ "P knows the secret", then

$$Pr[\overline{A}] = 1 - Pr[A] = 1 - (\frac{1}{2})^k$$

Outline

Graph 3-coloring is NP-complete:



Petersen graph

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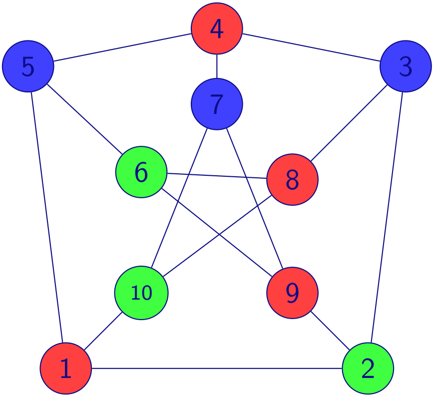
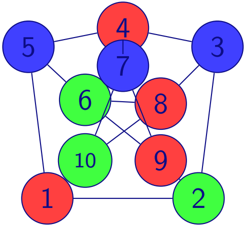


Petersen graph

P wants to prove to V his 3-coloring of $G = (E, V)$

P selects a permutation π of the 3 colors.

$\pi($



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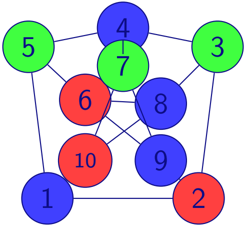

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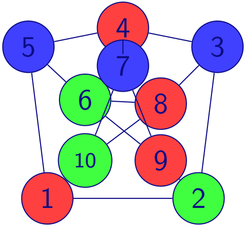
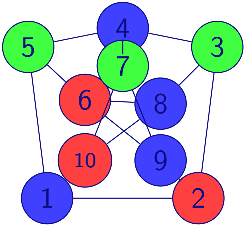
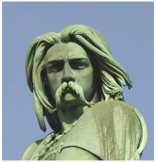
$\pi($ 

$)=$ 

Chooses $\forall u \in V, r_u$



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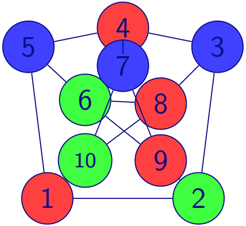

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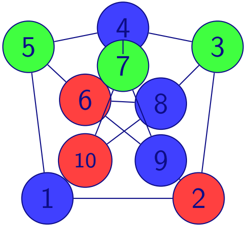
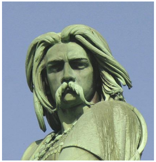


$\rightarrow \forall u \in V, e_u = H(\pi(c(u))||r_u) \rightarrow$



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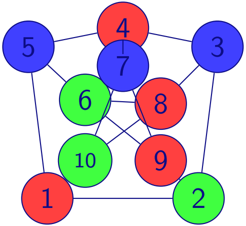
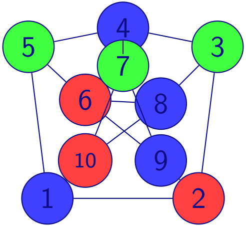
$\pi($ 

$)=$ 

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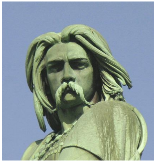

Chooses i and j

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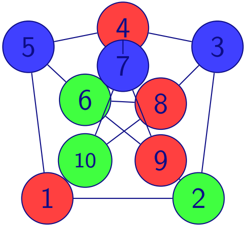

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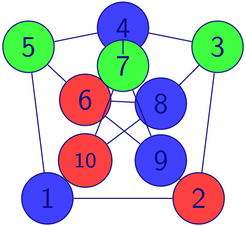

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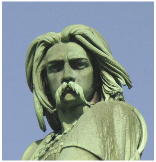

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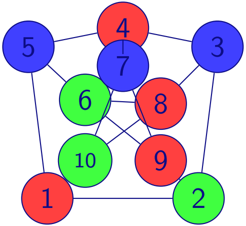
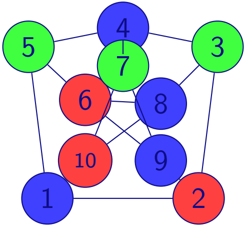
$\pi($ 

$)=$ 

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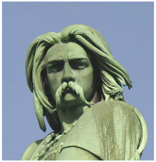

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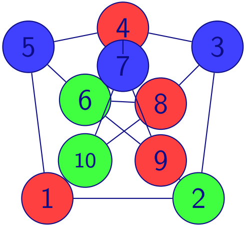

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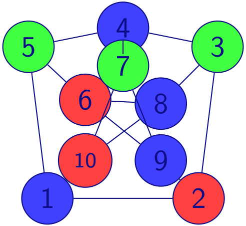

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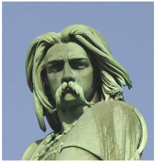
V accepts, if $e_{u_i} = H(\pi(c(u_i))||r_{u_i})$ and $e_{u_j} = H(\pi(c(u_j))||r_{u_j})$



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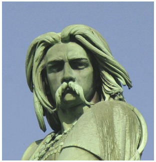
Outline

Schnorr Protocol, 1991

Let G_q a cyclic group of order q with a public generator g

Goal

P wants to prove the knowledge of x, where $y = g^x$



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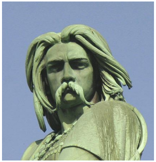

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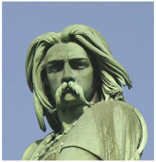

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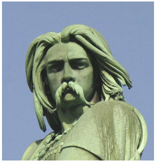

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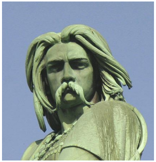

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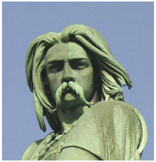
$$\longrightarrow t = g^r \longrightarrow$$

$$\longleftarrow c \longleftarrow$$

$$\longrightarrow s = r + x \cdot c \longrightarrow$$



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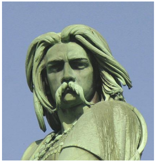
V accepts, if $t \cdot y^c = g^s$



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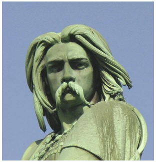

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V accepts, if $t \cdot y^c = g^s$

$$t \cdot y^c = g^r \cdot (g^x)^c = g^{r+x \cdot c} = g^s$$

Outline

Things to bring home

- Existence of Interactive Zero-knowledge Proof
- 3 protocols :
  1. Cave
  2. Graph 3 coloring
  3. Discret Logarithm (Schnorr)

Thank you for your attention.

Questions ?