Shaken, not Stirred: Automated Discovery of Subtle Attacks on Protocols using Mix-Nets

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Designing Secure Schemes is Difficult!

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- ✗ Publish the proof and wait until someone finds a mistake.
- ✓ Computer-Aided Security: ProVerif

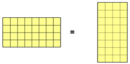
Shaken, not Stirred

Automated Discovery of Subtle Attacks on Protocols using Mix-Nets

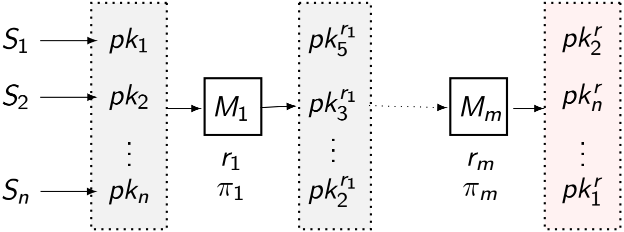
Shaken

Stirred

Exponentiation Mix-Nets Haenni *et al.* USENIX'11



El Gamal, $pk_i = g^{sk_i}$



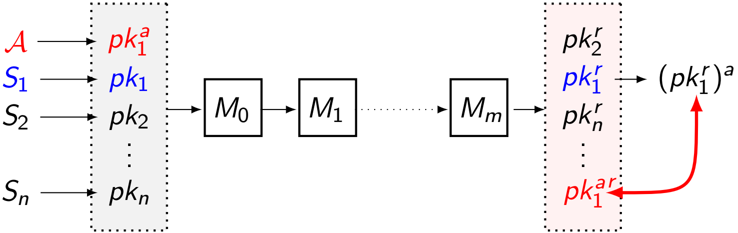
$$\pi = \prod_{i=1}^{m} \pi_i, \; r = \prod_{i=1}^{m} r_i$$

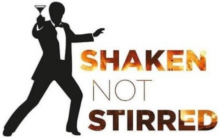
with sk_i, everyone can check $(g^r)^{sk_i} = pk_i^r$

For anonymity and unlinkability of voters

Shaken   Stirred

Attack Exponentiation Mix-Nets: Pfitzmann 1994, Rakeei *et al.* 2022





Attack against Re-encryption Mix-Nets Park et al. 1994 for voting

Candidates are public

$C_k = (g^{r_k}, m_k h^{r_k})$

Decryption by the vote authority

$$\mathcal{A}, a \rightarrow \widehat{C_k}$$
$$V_1, m_1 \rightarrow C_1$$
$$V_2, m_2 \rightarrow C_2$$
$$\vdots$$
$$V_k, m_k \rightarrow C_k$$
$$\vdots$$
$$V_n, m_n \rightarrow C_n$$

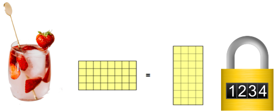
$$\rightarrow \boxed{M_0} \rightarrow \boxed{M_1} \cdots \rightarrow \boxed{M_m} \rightarrow$$

C'_2	m_2
C'_k	$m_k \rightarrow (m_k)^a$
C'_2	m_2
$\vdots$	$\vdots$
C'_n	m_n
$\vdots$	$\vdots$
$\widehat{C_k}'$	m_k^a

$$\widehat{C_k} = (C_k)^a = (g^{r_k a}, (m_k h^{r_k})^a)) = (g^{r_k a}, m_k{}^a h^{r_k a}))$$

Contributions

Can we find **automatically** such “cryptographic” attacks?



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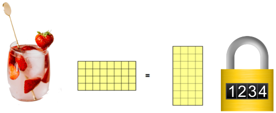
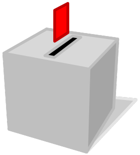


ProVerif models for Mixnet:

- Exponentiation
- ElGamal
- Weak and Strong NIZKP

Applications:



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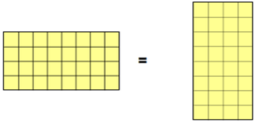
Exponentiation and Signature Modeling



Exponentiation

$$\begin{aligned}(g^x)^y &= (g^y)^x \\ ((g^x)^y)^z &= ((g^x)^z)^y \\ &= ((g^z)^x)^y \\ &= ((g^z)^y)^x \\ &= ((g^y)^z)^x \\ &= ((g^y)^x)^z\end{aligned}$$

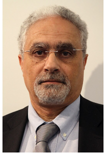
$$exp(exp(g, x), y) = exp(exp(g, y), x)$$
$$exp(exp(exp(g, x), y), z) = exp(exp(exp(g, x), z), y)$$

Signature

$pk = g^{sk}$, $\sigma = sign(m, g, sk)$, $checksign(\sigma, pk) = m$

$$getmess(sign(m, X, sk)) = m$$
$$checksign(sign(m, X, sk), X, exp(X, sk)) = m$$

ElGamal Encryption Modeling



ElGamal

- Encryption and decryption

$pk = g^{sk},\ c = (g^r, (g^{sk})^r m)$

$dec(enc(m, X, exp(X, sk), r), X, sk) = m$

- Re-Encryption with $g^{r'}$

$c' = (g^{r'} g^r, g^{r'} g^{sk} m)$

$reenc(enc(m, X, exp(X, sk), r), r', X, exp(X, sk)) = enc(m, X, exp(X, sk), sum(r, r'))$

$c^a = ((g^r)^a, (g^{sk})^a m^a)$

$EXP(enc(m, X, exp(X, x), r), a) = enc(exp(m, a), X, exp(X, x), mult(r, a))$

Non-Interactive ZKP (NIZKP)

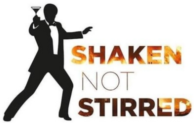
Weak NIZKP

Public parameter: $pk = g^{sk}$

- Construction with sk and a random
  $A = g^a$, $c = H(A)$, $f = a + c.sk$, $\pi = (c, f)$
- Verification of $\pi = (c, f)$ with pk, check $H(g^f \cdot pk^{-c}) \stackrel{?}{=} c$
  $H(g^f \cdot pk^{-c}) = H(g^{a+c \cdot sk} \cdot g^{-sk \cdot c}) = H(g^a) = c$

Fake a Weak NIZKP

- Construction with A' and f' two randoms
  $c' = H(A')$ and produce $\pi' = (c', f')$
- Verification for $pk' = (g^{f'} \cdot A'^{-1})^{c'^{-1}}$
  $H(g^{f'} \cdot pk'^{-c'}) = H(g^{f'} \cdot ((g^{f'} \cdot A'^{-1})^{c'^{-1}})^{-c'}) = H(A') = c'$



Allows attack against Exponentiation Mix-Nets with Weak NIZKP

Strong NIZKP: $c = H(A, pk)$

Non-Interactive ZKP

Weak NIZPK attack: Link of pk with $pk' = pk^{-c'^{-1}}$
$A' = g^{f'}.pk$ and $c' = H(A')$ then $\pi' = (c', f')$

Verification of $\pi' = (c', f')$ with pk', check $H(g^{f'}.pk'^{-c'}) \stackrel{?}{=} c'$:
$H(g^{f'}.pk'^{-c'}) = H(A'.pk^{-1}(pk^{-c'^{-1}})^{-c'}) = H(A'.pk^{-1}.pk) = H(A')$

ProVerif Modelling

- Weak ZKP: $c = H(A)$
  $$check(wzkp(A, X, sk), X, exp(X, sk), H(A)) = true$$

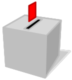
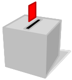
- Strong ZKP: $c = H(A, pk)$
  $$check(szkp(A, g, sk), g, exp(g, sk), H(A, g, exp(g, sk))) = true$$

Weak modeling allows the intruder to choose the value of the public key !

Results on Mixnets

Protocol	ZKP	Result	Time
Exponentiation Mix-Nets	without	✗	2 s
	weak	✗	1 m 6 s
	strong	✓	3 s
Re-encryption Mix-Nets	without	✗	1 s
	weak	✗	2 s
	strong	✓	1 s

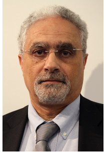
Applications

Protocol	ZKP	Property	Result	Time
Remark! 	without	Anonymous Marking	✗	3 m 16 s
		Anonymous Examiner	✗	4 m 19 s
	weak	Anonymous Marking	✗	9 m 35 s
		Anonymous Examiner	✗	9 m 23 s
	strong	Anonymous Marking	✓	11 s
		Anonymous Examiner	✓	7 s
Haenni Voting 	without	Vote Privacy	✗	4 m 35 s
	weak		✗	9 m 35 s
	strong		✓	14 s
	weak	Anonymous Shuffling	✗	4 m 6 s
	strong		✓	9 s
Estonian IVXV 	without	Vote Privacy	✗	1 s
	weak		✗	25 s
	strong		✓	8 s

Conclusion

New ProVerif models for:

- Exponentiation Mixnets
- Re-Encryption Mixnets
- Weak ZKP
- Strong ZKP
- ElGamal
- Signature



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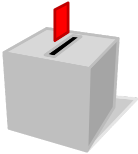

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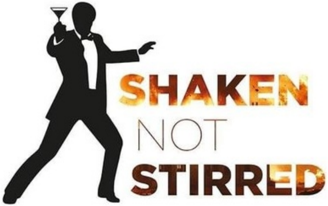

Thanks for your attention!