

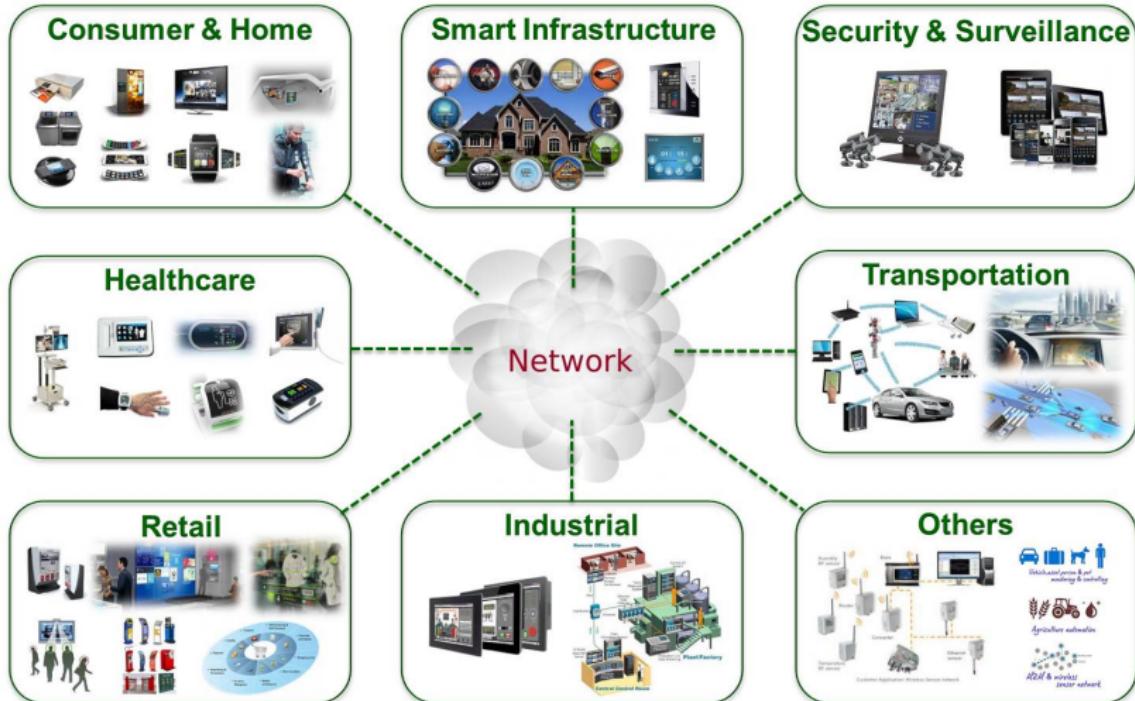
# Security of IoT

Pascal Lafourcade



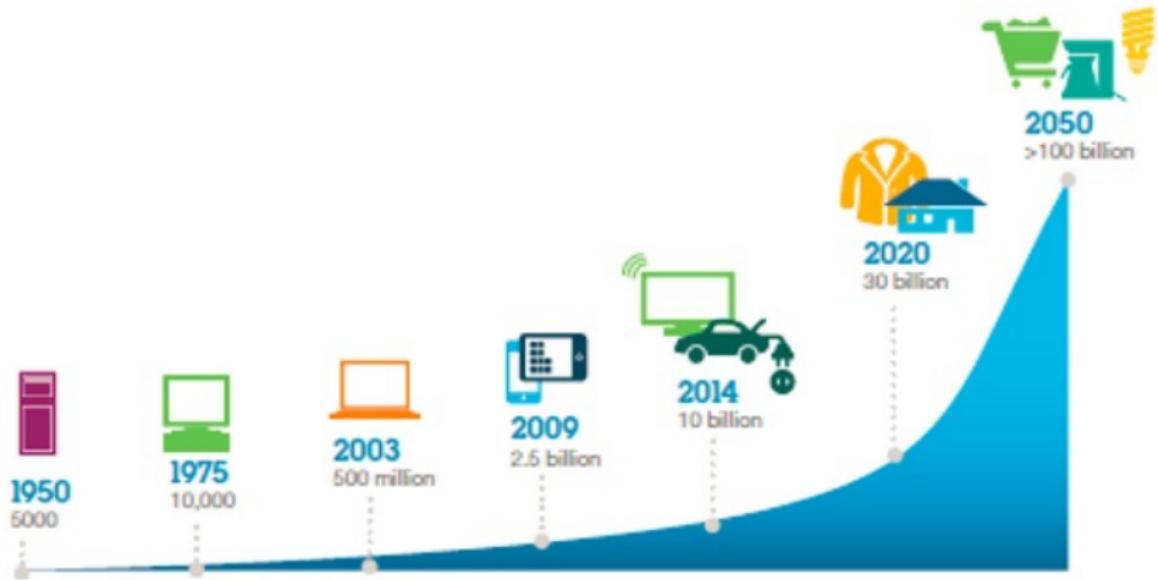
October 2020

# Internet of Thing (IoT)



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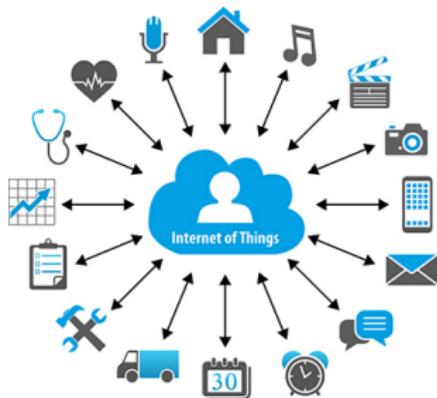
# Increasing Success of IoT



# Reasons of the Success of IOT

## Technology

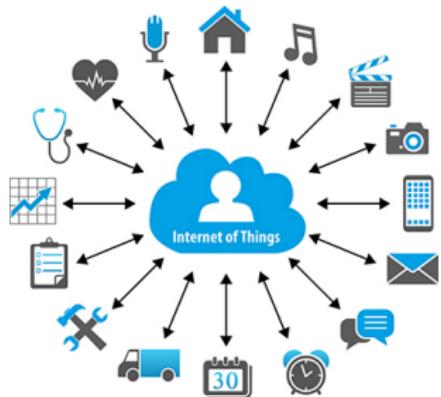
- ▶ Wireless Communications:  
Wifi, 3G, 4G, Bluetooth, Sigfox ...
- ▶ Batteries
- ▶ CPU
- ▶ Sensors
- ▶ Price



# Reasons of the Success of IOT

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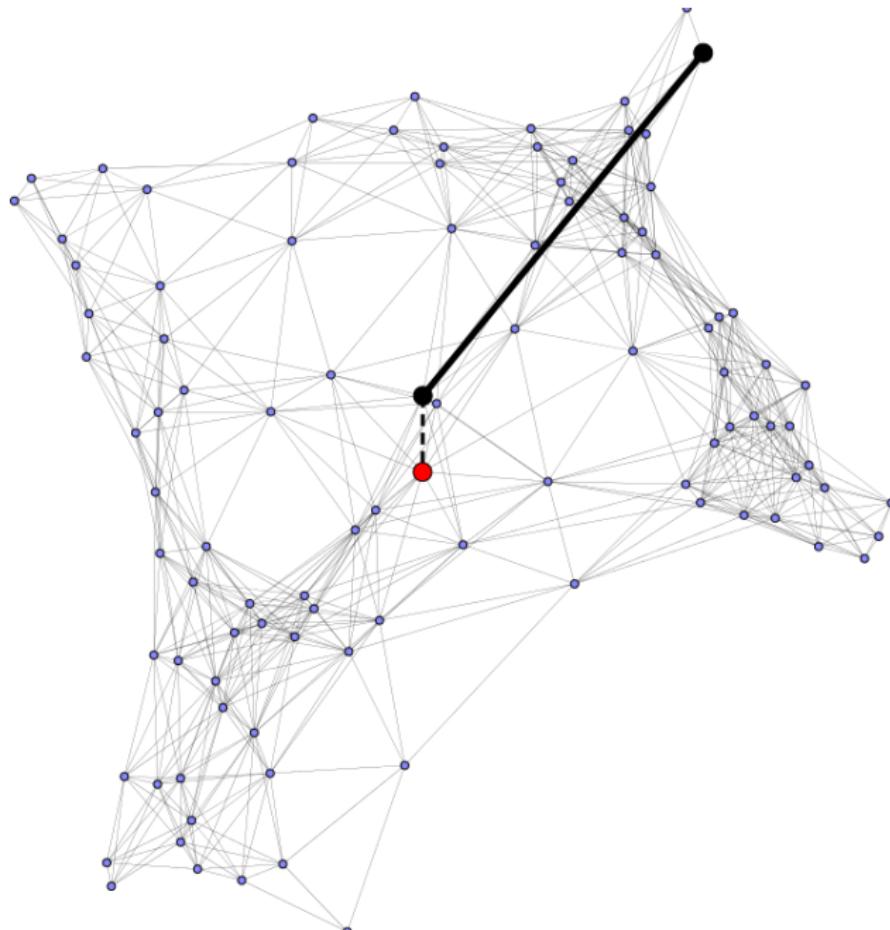
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## Usage

- ▶ Monitoring services
- ▶ Hyperconnectivity
- ▶ Availability

# Wireless communications $\Rightarrow$ Wormhole Attack



## Real attacks on IoT from 2007 ...



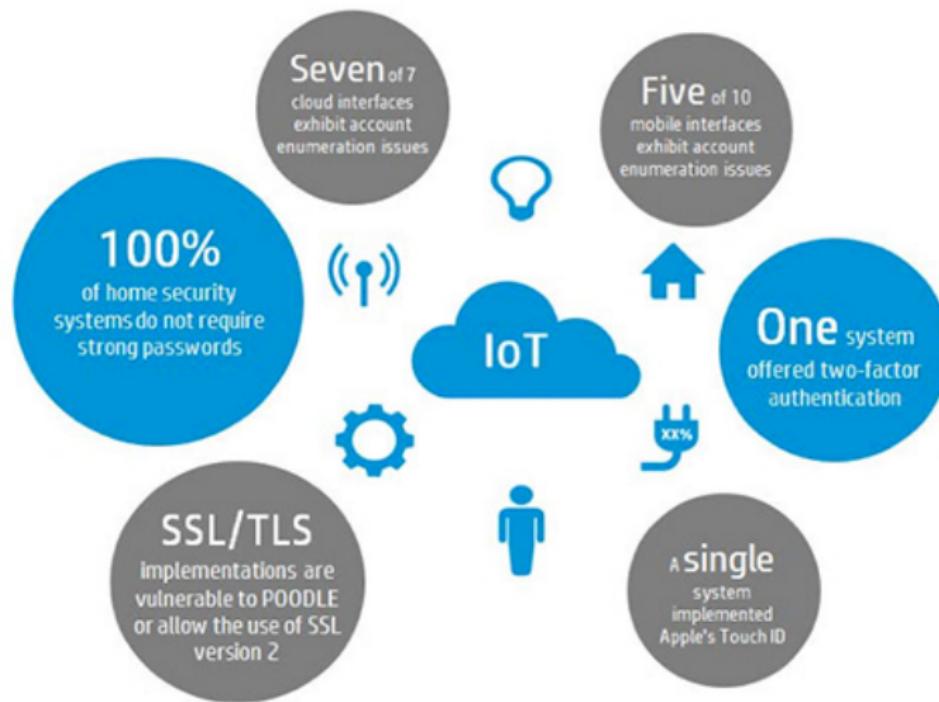
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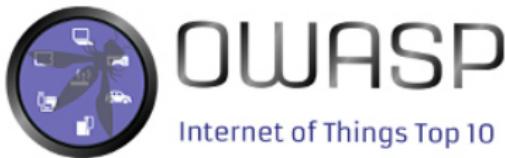


# Insecurity of IoT by HP in 2015



POODLE: Padding Oracle On Downgraded Legacy Encryption

# TOP 10: Vulnerabilities of IoT



1. Insecure Web Interface (weak passwords, account protection)
2. Unsufficient Authentication/Authorization
3. Insecure Network Services (ports open, DoS)
4. Lack of Transport Encryption
5. Privacy Concerns (leak of personal informations)
6. Insecure Cloud interfaces
7. Insecure Mobile Interfaces
8. Insufficient Security Configurability
9. Insecure Software/Firmware
10. Poor Physical Security

<https://www.owasp.org/images/8/8e/Infographic-v1.jpg>

# How to Secure IoT

## Cryptography:



- ▶ Primitives: RSA, Elgamal, AES, DES, SHA-3 ...
- ▶ Protocols: Distributed Algorithms

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TOP  
SECRET

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- ▶ Passive, active
- ▶ CPA, CCA ...

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TOP  
SECRET

## Intruders:



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Designing such **secure** protocols is **difficult**

# Is it preserving your privacy?



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4096 RSA encryption

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Environ 60 températures possibles: 35 ... 41

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4096 RSA encryption

Environ 60 températures possibles: 35 ... 41

$$\{35\}_{pk}, \{35, 1\}_{pk}, \dots, \{41\}_{pk}$$

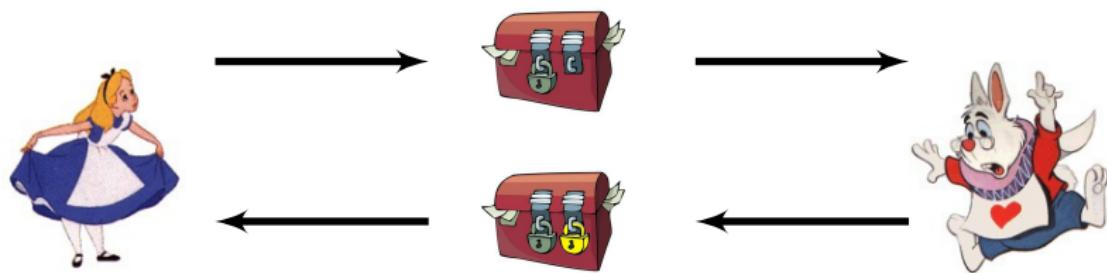
# 3-Pass Shamir



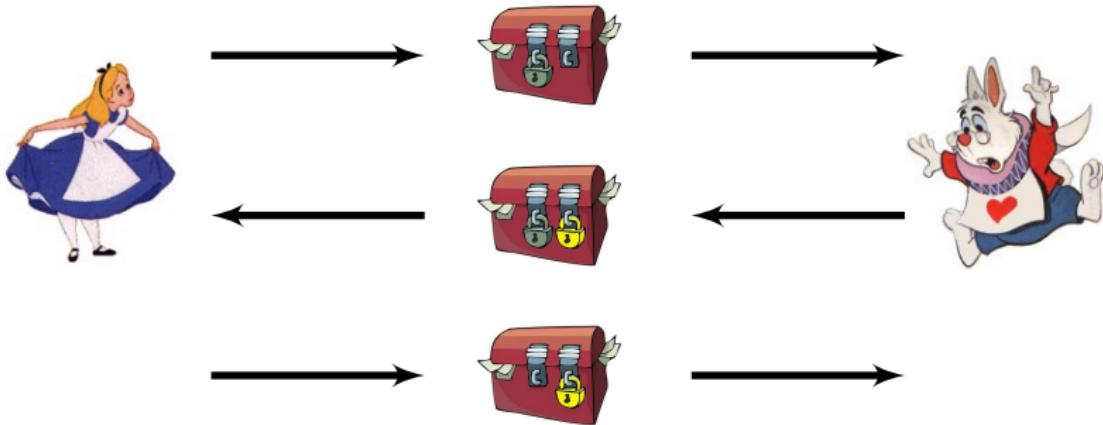
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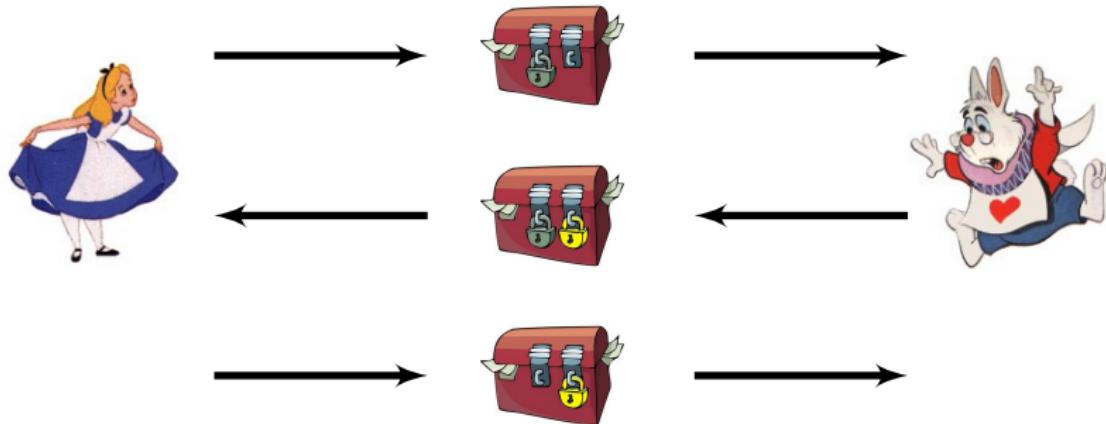
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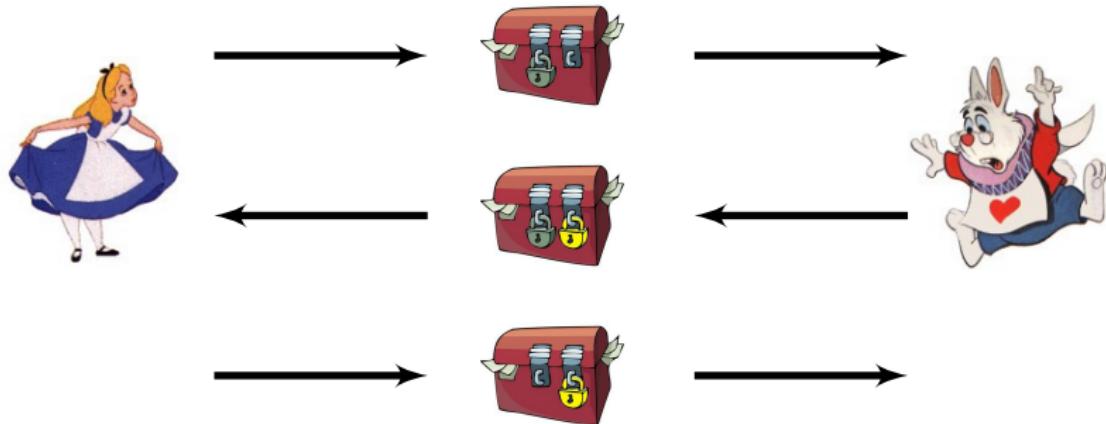
## 3-Pass Shamir



## Abstract Representation

$$1 \ A \rightarrow B : \{m\}_{K_A}$$

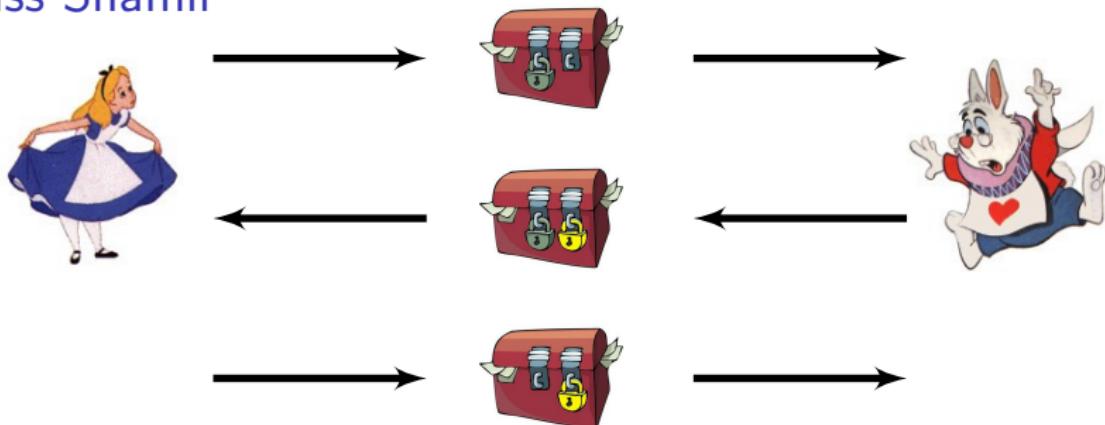
## 3-Pass Shamir



## Abstract Representation

$$\begin{array}{l} 1 \quad A \rightarrow B : \{m\}_{K_A} \\ 2 \quad B \rightarrow A : \{\{m\}_{K_A}\}_{K_B} \end{array}$$

## 3-Pass Shamir



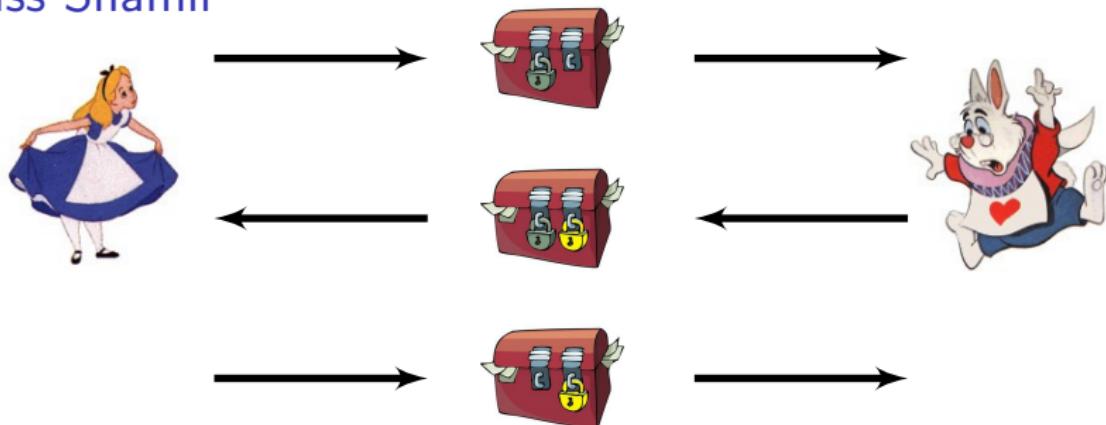
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$$1 \quad A \rightarrow B : \{m\}_{K_A}$$

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Commutative  
Encryption

## 3-Pass Shamir



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$$3 \quad A \rightarrow B : \{m\}_{K_B}$$

Commutative  
Encryption

# Logical Attack on Shamir 3-Pass Protocol (I)

Perfect encryption one-time pad (Vernam Encryption)

$$\{m\}_k = m \oplus k$$

## XOR Properties (ACUN)

- ▶  $(x \oplus y) \oplus z = x \oplus (y \oplus z)$  **Associativity**
- ▶  $x \oplus y = y \oplus x$  **Commutativity**
- ▶  $x \oplus 0 = x$  **Unity**
- ▶  $x \oplus x = 0$  **Nilpotency**

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- ▶  $x \oplus 0 = x$  **U**nity
- ▶  $x \oplus x = 0$  **N**ilpotency

Vernam encryption is a **commutative encryption** :

$$\{\{m\}_{K_A}\}_{K_I} = (m \oplus K_A) \oplus K_I = (m \oplus K_I) \oplus K_A = \{\{m\}_{K_I}\}_{K_A}$$

## Logical Attack on Shamir 3-Pass Protocol (II)

Perfect encryption one-time pad (Vernam Encryption)

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Shamir 3-Pass Protocol



- 1    $A \rightarrow B : m \oplus K_A$
- 2    $B \rightarrow A : (m \oplus K_A) \oplus K_B$
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Shamir 3-Pass Protocol



- 1    $A \rightarrow B : m \oplus K_A$
- 2    $B \rightarrow A : (m \oplus K_A) \oplus K_B$
- 3    $A \rightarrow B : m \oplus K_B$

## Second Example

Needham Schroeder Key Exchange 1976

$$A \rightarrow B : \{A, N_A\}_{Pub(B)}$$
$$B \rightarrow A : \{N_A, N_B\}_{Pub(A)}$$
$$A \rightarrow B : \{N_B\}_{Pub(B)}$$

- ▶ Use cryptography
- ▶ Small programs
- ▶ Distributed

# Cryptography is not sufficient !

Example : Needham Schroeder Key Exchange

$A \rightarrow B : \{A, N_A\}_{Pub(B)}$

$B \rightarrow A : \{N_A, N_B\}_{Pub(A)}$

$A \rightarrow B : \{N_B\}_{Pub(B)}$

# Cryptography is not sufficient !

Example : Needham Schroeder Key Exchange

$$A \rightarrow B : \{A, N_A\}_{Pub(B)}$$
$$B \rightarrow A : \{N_A, N_B\}_{Pub(A)}$$
$$A \rightarrow B : \{N_B\}_{Pub(B)}$$

Broken 17 years after, by G. Lowe

$$A \rightarrow I : \{A, N_A\}_{Pub(I)}$$
$$I \rightarrow B : \{A, N_A\}_{Pub(B)}$$
$$A \leftarrow I : \{N_A, N_B\}_{Pub(A)}$$
$$I \leftarrow B : \{N_A, N_B\}_{Pub(A)}$$
$$A \rightarrow I : \{N_B\}_{Pub(I)}$$
$$I \rightarrow B : \{N_B\}_{Pub(B)}$$

# Security Challenges for IoT

Data exchanged should be protected.

## Security Properties

- ▶ Data Integrity
- ▶ Data Confidentiality
- ▶ Data Privacy
- ▶ Authentication
- ▶ Non-repudiation
- ▶ Availability



# Outline

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Classical Asymmetric Encryptions

Elliptic Curves

Classical Symetric Encryptions

DES

3-DES

AES

Modes

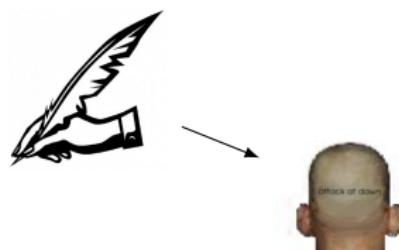
Hash Functions

Conclusion

# Information hiding

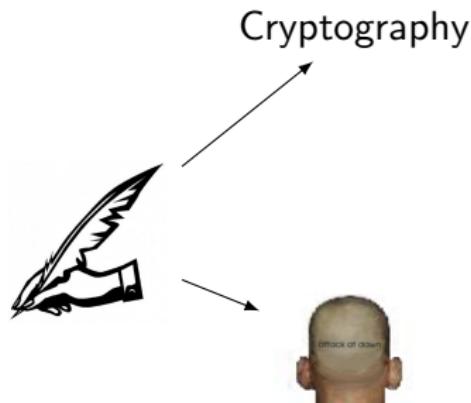


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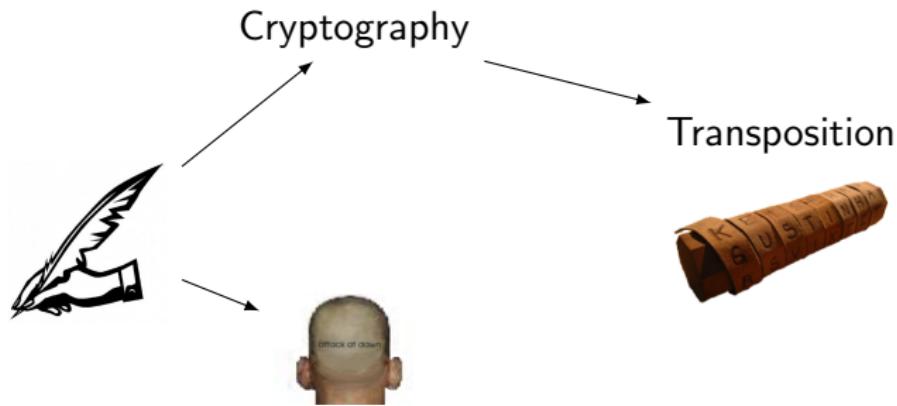
Stéganographie

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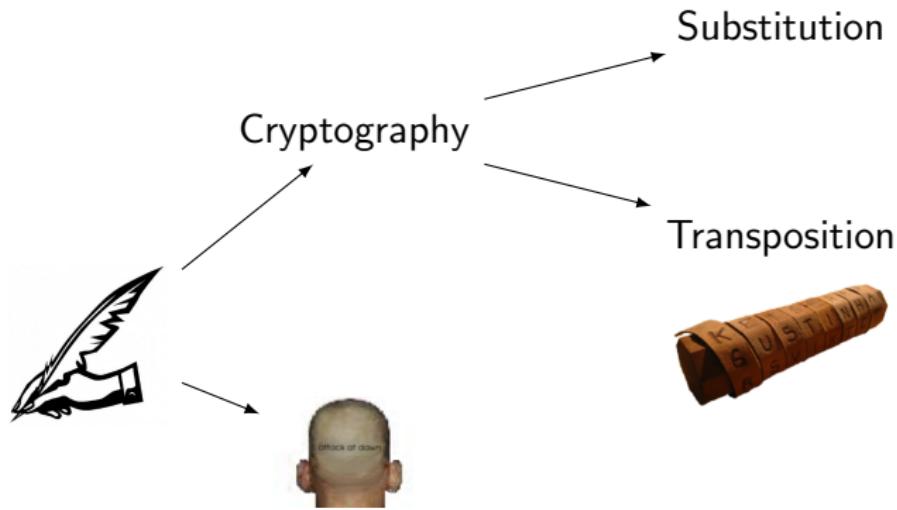
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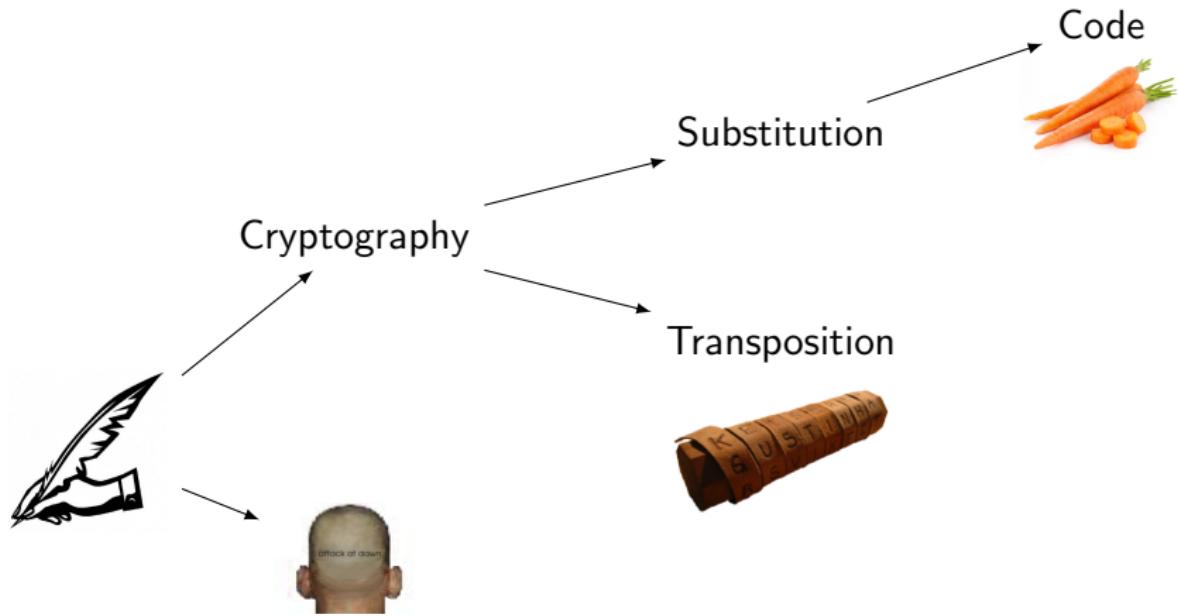


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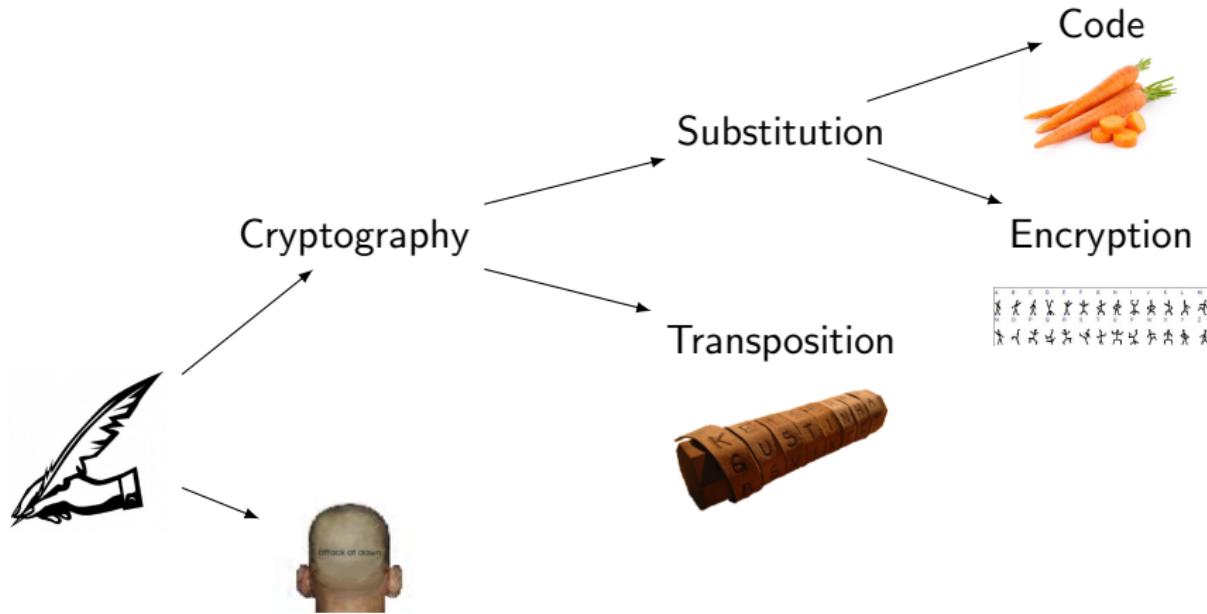


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Stéganographie

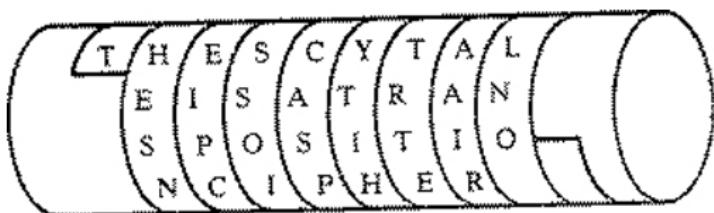
# Information hiding



# Greeks and the Scythale



# Greeks and the Scythale



Transposition

## Transposition ciphers

- ▶ For block length  $t$ , let  $K$  be the set of permutations on  $\{1, \dots, t\}$ . For each  $e \in K$  and  $m \in M$

$$E_e(m) = m_{e(1)}m_{e(2)} \cdots m_{e(t)}.$$

- ▶ The set of all such transformations is called a **transposition cipher**.
- ▶ To decrypt  $c = c_1c_2 \cdots c_t$  compute  $D_d(c) = c_{d(1)}c_{d(2)} \cdots c_{d(t)}$ , where  $d$  is inverse permutation.
- ▶ Letters unchanged so frequency analysis can be used to reveal if ciphertext is a transposition. Decrypt by exploiting frequency analysis for diphthongs, triphthongs, words, etc.

# Romans



Caesar Encryption  
Substitution +3

# Romans



Caesar Encryption  
Substitution +3

Dyh Fhvdu

# Romans



Caesar Encryption  
Substitution +3

Dyh Fhvdu

Ave Cesar

## Mono-alphabetic substitution ciphers

- ▶ Simplest kind of cipher. Idea over 2,000 years old.
- ▶ Let  $K$  be the set of all permutations on the alphabet  $A$ . Define for each  $e \in K$  an encryption transformation  $E_e$  on strings  $m = m_1 m_2 \cdots m_n \in M$  as

$$E_e(m) = e(m_1)e(m_2) \cdots e(m_n) = c_1 c_2 \cdots c_n = c.$$

- ▶ To decrypt  $c$ , compute the inverse permutation  $d = e^{-1}$  and

$$D_d(c) = d(c_1)d(c_2) \cdots d(c_n) = m.$$

- ▶  $E_e$  is a simple substitution cipher or a mono-alphabetic substitution cipher.

## Substitution cipher examples

- ▶ KHOOR ZRUOG

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- ▶ KHOOR ZRUOG = HELLO WORLD

**Caesar cipher:** each plaintext character is replaced by the character three to the right modulo 26.

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**ROT13:** shift each letter by 13 places.

Under Unix: `tr a-zA-Z n-za-mN-ZA-M.`

- ▶ 2-25-5 2-25-5

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- ▶ 2-25-5 2-25-5 = BYE BYE

**Alphanumeric:** substitute numbers for letters.

How hard are these to cryptanalyze? Caesar? General?

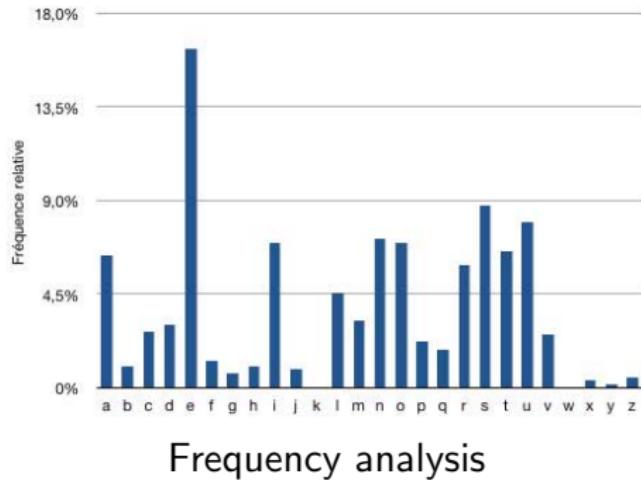
Is it secure?

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Key spaces are typically huge. 26 letters  $\rightsquigarrow$  26! possible keys.

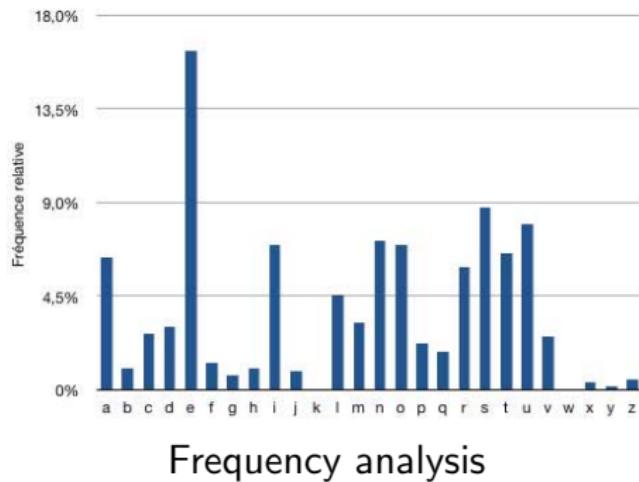
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Except for short, atypical texts

*From Zanzibar to Zambia and Zaire, ozone zones make zebras run  
zany zigzags.*

⇒ More sophistication required to mask statistical regularities

## Homophonic substitution ciphers

- ▶ To each  $a \in A$ , associate a set  $H(a)$  of strings of  $t$  symbols, where  $H(a), a \in A$  are pairwise disjoint. A **homophonic substitution cipher** replaces each  $a$  with a randomly chosen string from  $H(a)$ . To decrypt a string  $c$  of  $t$  symbols, one must determine an  $a \in A$  such that  $c \in H(a)$ . The key for the cipher is the sets  $H(a)$ .

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### Example:

$A = \{a, b\}$ ,  $H(a) = \{00, 10\}$ , and  $H(b) = \{01, 11\}$ . The plaintext  $ab$  encrypts to one of 0001, 0011, 1001, 1011.

Rational: makes frequency analysis more difficult.

Cost: data expansion and more work for decryption.

# Polyalphabetic substitution ciphers

- ▶ Leon Alberti: conceal distribution using family of mappings.



- ▶ A **polyalphabetic substitution cipher** is a block cipher with block length  $t$  over alphabet  $A$  where:
  - ▶ the key space  $K$  consists of all ordered sets of  $t$  permutations over  $A$ ,  $(p_1, p_2, \dots, p_t)$ .
  - ▶ Encryption of  $m = m_1 \dots m_t$  under key  $e = (p_1, \dots, p_t)$  is  $E_e(m) = p_1(m_1) \dots p_t(m_t)$ .
  - ▶ Decryption key for  $e$  is  $d = (p_1^{-1}, \dots, p_t^{-1})$ .

## Example: Vigenère ciphers 1553

- ▶ Key given by sequence of numbers  $e = e_1, \dots, e_t$ , where

$$p_i(a) = (a + e_i) \bmod n$$

defining a permutation on an alphabet of size  $n$ .

- ▶ Example: English ( $n = 26$ ), with  $k = 3, 7, 10$

$m = \text{THI SCI PHE RIS CER TAI NLY NOT SEC URE}$

then

$E_e(m) = \text{WOS VJS SOO UPC FLB WHS QSI QVD VLM XYO}$

## One-time pads (Vernam cipher)

- ▶ A **one-time pad** is a cipher defined over  $\{0, 1\}$ . Message  $m_1 \dots m_n$  is encrypted by a binary key string  $k_1 \dots k_n$ .

$$E_{k_1 \dots k_n}(m_1 \dots m_n) = (m_1 \oplus k_1) \dots (m_n \oplus k_n)$$

$$D_{k_1 \dots k_n}(c_1 \dots c_n) = (c_1 \oplus k_1) \dots (c_n \oplus k_n)$$

- ▶ Unconditional (information theoretic) security,  
if key isn't reused!



# One-Time Pad (Vernam 1917)



Example:

$$\begin{array}{r} m = 010111 \\ k = 110010 \\ \hline c = 100101 \end{array}$$

Problem?

## One-Time Pad (Vernam 1917)



Example:

$$\begin{array}{r} m = 010111 \\ k = 110010 \\ \hline c = 100101 \end{array}$$

Problem? Securely exchanging and synchronizing long keys.

## Kerchoff's Principle

In 1883, a Dutch linguist Auguste Kerchoff von Nieuwenhof stated in his book “La Cryptographie Militaire” that:

“the security of a crypto-system must be totally dependent on the secrecy of the key, not the secrecy of the algorithm.”

Author's name sometimes spelled Kerckhoff

# Chiffrement : Enigma (Seconde guerre mondiale)



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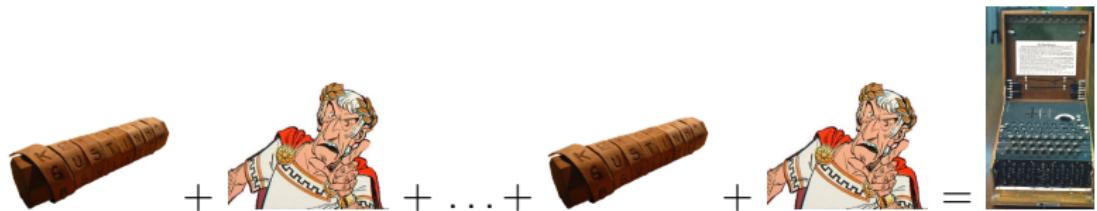
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## Shannon's Principle 1949

### Confusion

The purpose of confusion is to make the relation between the key and the ciphertext as complex as possible.

Ciphers that do not offer much confusion (such as Vigenere cipher) are susceptible to frequency analysis.

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## Diffusion

Diffusion spreads the influence of a single plaintext bit over many ciphertext bits.

The best diffusing component is substitution (homophonic)

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## Principle

A good cipher design uses Confusion and Diffusion together

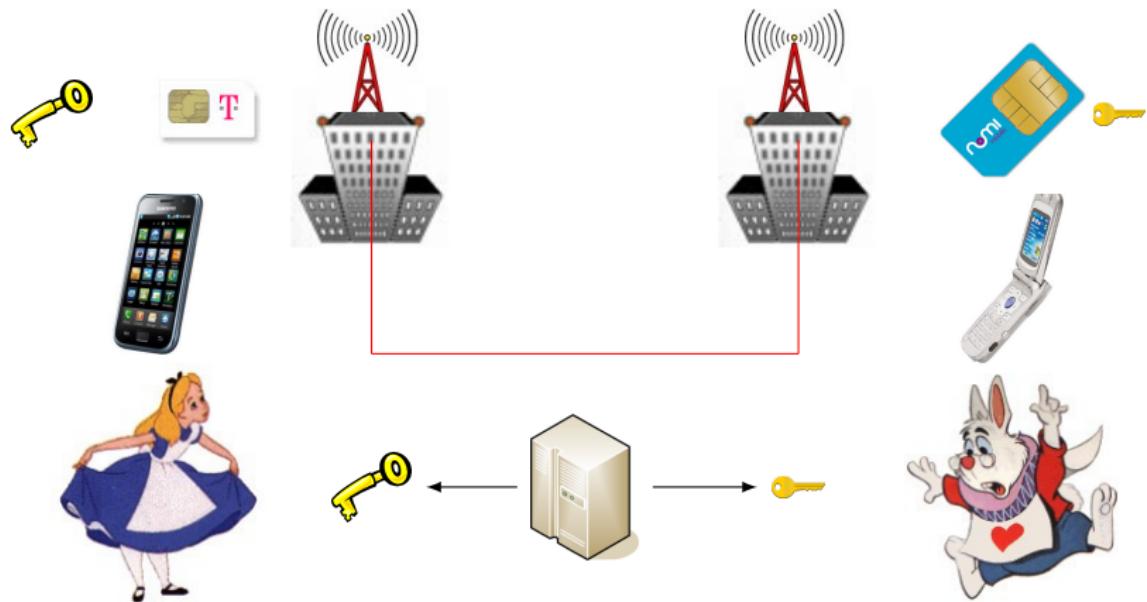
# Symmetric Encryption



## Examples

- ▶ DES
- ▶ AES

# Cellphone Communications



# Public Key Encryption



## Examples

- ▶ RSA :  $c = m^e \pmod n$
- ▶ ElGamal :  $c \equiv (g^r, h^r \cdot m)$

## Comparison

- ▶ Size of the key
- ▶ Complexity of computation (time, hardware, cost ...)
- ▶ Number of different keys ?
- ▶ Key distribution
- ▶ Signature only possible with asymmetric scheme

## Computational cost of encryption

2 hours of video (assumes 3Ghz CPU)

| Schemes     | DVD 4,7 G.B |         | Blu-Ray 25 GB |         |
|-------------|-------------|---------|---------------|---------|
|             | encrypt     | decrypt | encrypt       | decrypt |
| RSA 2048(1) | 22 min      | 24 h    | 115 min       | 130 h   |
| RSA 1024(1) | 21 min      | 10 h    | 111 min       | 53 h    |
| AES CTR(2)  | 20 sec      | 20 sec  | 105 sec       | 105 sec |

# Outline

History of Cryptography

Classical Asymmetric Encryptions

Elliptic Curves

Classical Symetric Encryptions

DES

3-DES

AES

Modes

Hash Functions

Conclusion

# One-way function and Trapdoor

## Definition

A function is *One-way*, if :

- ▶ it is easy to compute
- ▶ its inverse is hard to compute :

$$\Pr[m \xleftarrow{r} \{0, 1\}^*; y := f(m) : f(A(y, f)) = y]$$

is negligible.

Trapdoor:

- ▶ Inverse is easy to compute given an additional information (an inverse key e.g. in RSA).

# Integer Factoring

→ Use of algorithmically hard problems.

## Factorization

- ▶  $p, q \mapsto n = p \cdot q$     easy (quadratic)
- ▶  $n = p \cdot q \mapsto p, q$     difficult

# RSA

RSA function  $n = pq$ ,  $p$  and  $q$  primes.

e: public exponent

- ▶  $x \mapsto x^e \pmod{n}$  easy (cubic)
- ▶  $y = x^e \mapsto x \pmod{n}$  difficult  
 $x = y^d$  where  $d = e^{-1} \pmod{\phi(n)}$

## Soundness

Assume  $n = pq$ ,  $\gcd(e, \phi(n)) = 1$  and  $d = e^{-1} \pmod{\phi(n)}$ .

$$c^d = m^{de} = m \cdot m^{k\phi(n)} \pmod{n}$$

According to the Fermat Little Theorem  $\forall x \in (\mathbb{Z}/n\mathbb{Z})^*, x^{\phi(n)} = 1$

## Example RSA

### Example

- ▶  $p = 61$  (destroy this after computing E and D)
- ▶  $q = 53$  (destroy this after computing E and D)
- ▶  $n = pq = 3233$  modulus (give this to others)
- ▶  $e = 17$  public exponent (give this to others)
- ▶  $d = 2753$  private exponent (keep this secret!)

Your public key is  $(e, n)$  and your private key is  $d$ .

$$\text{encrypt}(T) = (T^e) \bmod n = (T^{17}) \bmod 3233$$

$$\text{decrypt}(C) = (C^d) \bmod n = (C^{2753}) \bmod 3233$$

- ▶  $\text{encrypt}(123) = 123^{17} \bmod 3233$   
 $= 337587917446653715596592958817679803 \bmod 3233$   
 $= 855$
- ▶  $\text{decrypt}(855) = 855^{2753} \bmod 3233$

# Complexity Estimates

Estimates for integer factoring Lenstra-Verheul 2000

| Modulus<br>(bits) | Operations<br>( $\log_2$ ) |
|-------------------|----------------------------|
| 512               | 58                         |
| 1024              | 80                         |
| 2048              | 111                        |
| 4096              | 149                        |
| 8192              | 156                        |

$\approx 2^{60}$  years

→ Can be used for RSA too.

## ElGamal Encryption Scheme

**Key generation:** Alice chooses a prime number  $p$  and a group generator  $g$  of  $(\mathbb{Z}/p\mathbb{Z})^*$  and  $a \in (\mathbb{Z}/(p-1)\mathbb{Z})^*$ .

**Public key:**  $(p, g, h)$ , where  $h = g^a \pmod{p}$ .

**Private key:**  $a$

**Encryption:** Bob chooses  $r \in_R (\mathbb{Z}/(p-1)\mathbb{Z})^*$  and computes  $(u, v) = (g^r, Mh^r)$

**Decryption:** Given  $(u, v)$ , Alice computes  $M \equiv_p \frac{v}{u^a}$

**Justification:**  $\frac{v}{u^a} = \frac{Mh^r}{g^{ra}} \equiv_p M$

**Remarque:** re-use of the same random  $r$  leads to a security flaw:

$$\frac{M_1 h^r}{M_2 h^r} \equiv_p \frac{M_1}{M_2}$$

**Practical Inconvenience:** Cipher is twice as long as plain text.

## Optimal Asymmetric Encryption Padding (OAEP)

The OAEP cryptosystem  $(K, E, D)$  obtained from a permutation  $f$ , whose inverse is denoted by  $g$ . And two hash functions:

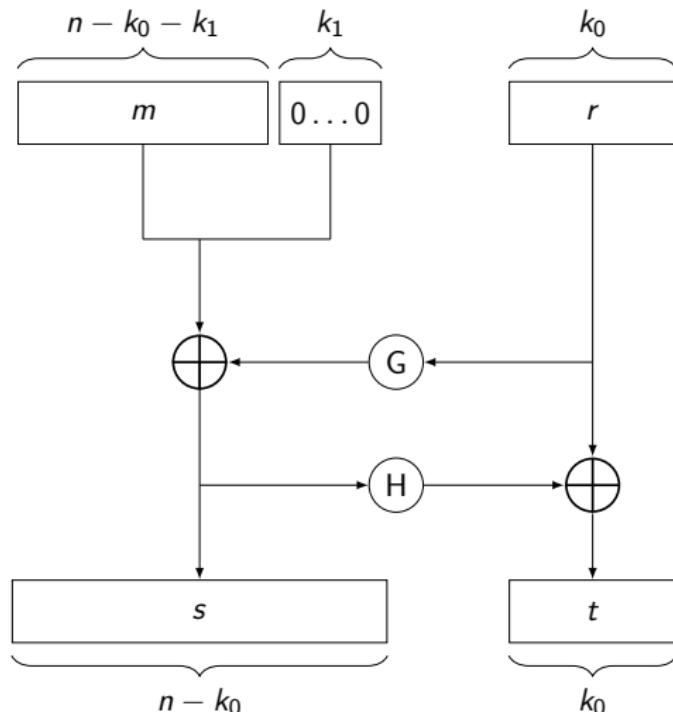
$$G : \{0, 1\}^{k_0} \rightarrow \{0, 1\}^{k-k_0}$$

$$H : \{0, 1\}^{k-k_0} \rightarrow \{0, 1\}^{k_0}$$

$K(1^k)$ : specifies an instance of the function  $f$ , and of its inverse  $g$ .  
The public key  $pk$  is therefore  $f$  and the private key  $sk$  is  $g$ .

## OAEP: Encryption

$$E_{pk}(m, r) = c = f(s, t) \text{ with } m \in \{0, 1\}^n, \text{ and } r \leftarrow \{0, 1\}^{k_0}$$
$$s = (m||0^{k_1}) \oplus G(r), t = r \oplus H(s)$$



## OAEP: Decryption

$$D_{sk}(c)$$

$$g(c) = (s, t)$$

$$r = t \oplus H(s)$$

$$M = s \oplus G(r)$$

If  $[M]_{k_1} = 0^{k_1}$ , the algorithm returns  $[M]^n$ , otherwise it returns “Reject”

- ▶  $[M]_{k_1}$  denotes the  $k_1$  least significant bits of  $M$
- ▶  $[M]^n$  denotes the  $n$  most significant bits of  $M$

## Others Cryptosystems

Bellare & Rogaway'93:

$$f(r) \parallel x \oplus G(r) \parallel H(x \parallel r)$$

- ▶ Zheng & Seberry'93:

$$f(r) \parallel G(r) \oplus (x \parallel H(x))$$

- ▶ OAEP'94 (Bellare & Rogaway):

$$f(s \parallel r \oplus H(s))$$

where  $s = x0^k \oplus G(r)$

- ▶ OAEP+'02 (Shoup):

$$f(s \parallel r \oplus H(s))$$

where  $s = x \oplus G(r) \parallel H'(r \parallel x)$ .

- ▶ Fujisaki & Okamoto'99:

$$E((x \parallel r); H(x \parallel r))$$

where  $E$  is IND-CPA.

# Outline

History of Cryptography

Classical Asymmetric Encryptions

Elliptic Curves

Classical Symetric Encryptions

DES

3-DES

AES

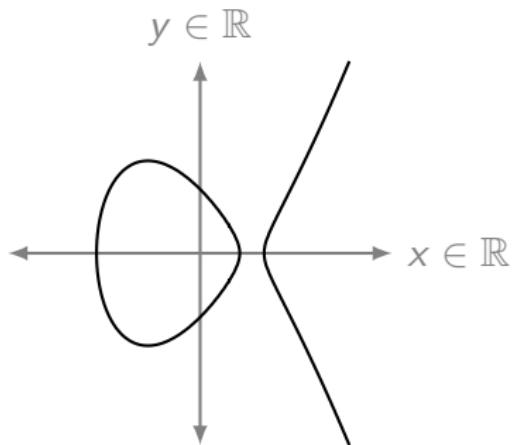
Modes

Hash Functions

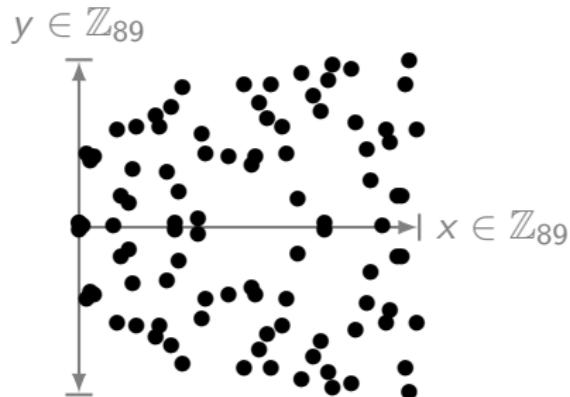
Conclusion

# Introduction

$$y^2 = x^3 + ax + b$$



$$y^2 = x^3 - 2x + 1 \text{ over } \mathbb{R}$$



$$y^2 = x^3 - 2x + 1 \text{ over } \mathbb{Z}_{89}$$

$E(K) = \{(x, y) \text{ such that } y^2 = x^3 + ax + b\}$  plus an extra point  
“at infinite”

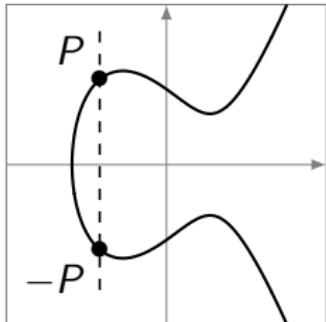
Weierstrass form if  $\Delta = -16(4a^3 + 27b^2) \neq 0$  (if K is not of characteristic 2 or 3).

# Laws

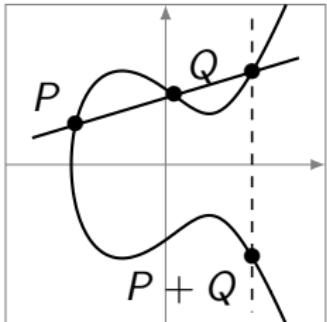
## Theorem

- ▶ Addition law on  $E(K)$ 
  - ▶ Associativity:  $(P_1 + P_2) + P_3 = P_1 + (P_2 + P_3)$
  - ▶ Commutativity:  $P_1 + P_2 = P_2 + P_1$
  - ▶ Neutral element is  $\infty$ :  $P + \infty = P$
  - ▶ Inverse: Given  $P$  on  $E$ , there exists  $P'$  on  $E$  with  $P + P' = \infty$  (usually denoted  $-P$ )
- ▶ Three aligned points sum to neutral element often denoted zero

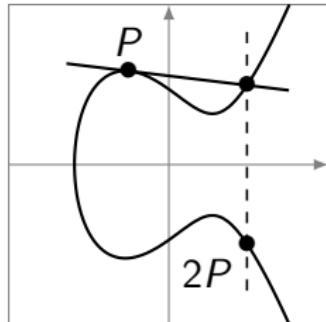
# Laws



Inverse element  $-P$



Addition  $P + Q$   
“Chord rule”



Doubling  $P + P$   
“Tangent rule”

$$P + R + Q = 0 \Rightarrow R = -(P + Q)$$

$$R + S + 0 = 0 \Rightarrow R = -S$$

## “Elliptic Discrete Logarithm”

### Hard Problem

Finding  $k$ , given  $P$  and  $Q = kP$ . is computationally intractable for large values of  $k$ .

## Cryptosystem: ECDH

Exercice: Give an Elliptic curve of ElGamal.

## Cryptosystem: ECDH

Alice's key is  $(d_A, Q_A)$  where  $Q_A = d_A G$ .

### DH like Protocol

1. Alice sends  $Q_A, G$  to Bob.
2. Bob computes  $k = d_B Q_A$ .
3. Bob sends to Alice  $Q_B$
4. Alice computes  $k = d_A Q_B$ .

The shared key is  $x_k$  (the  $x$  coordinate of the point).

The number calculated by both parties is equal, because  
 $k = d_A Q_B = d_A d_B G = d_B d_A G = d_B Q_A = k$ .

# ECDSA (Digital Signature Algorithm) I

Alice private key  $d_A$  and a public key  $Q_A$  (where  $Q_A = d_A G$ ).

## Signature generation algorithm

1. Calculate  $e = \text{HASH}(m)$ , where HASH is a cryptographic hash function, such as SHA-1.
2. Select a random integer  $k$  from  $[1, n - 1]$ .
3. Calculate  $r = x_1 \pmod{n}$ , where  $(x_1, y_1) = kG$ .  
If  $r = 0$ , go back to step 2.
4. Calculate  $s = k^{-1}(e + rd_A) \pmod{n}$ .  
If  $s = 0$ , go back to step 2.
5. The signature is the pair  $(r, s)$ .

# ECDSA (Digital Signature Algorithm) II

## Signature verification algorithm

1. Verify that  $r$  and  $s$  are integers in  $[1, n - 1]$ .  
If not, the signature is invalid.
2. Calculate  $e = \text{HASH}(m)$ , where HASH is the same function used in the signature generation.
3. Calculate  $w = s^{-1} \pmod{n}$ .
4. Calculate  $u_1 = ew \pmod{n}$  and  $u_2 = rw \pmod{n}$ .
5. Calculate  $(x_1, y_1) = u_1 G + u_2 Q_A$ .
6. The signature is valid if  $r = x_1 \pmod{n}$ , invalid otherwise.

## ECDSA (Digital Signature Algorithm)

$$s = k^{-1}(e + rd_A) \pmod{n}$$

Hence

$$\begin{aligned}k &= s^{-1}(e + rd_A) \pmod{n} = w(e + rd_A) = we + wrd_A = u_1 + u_2 d_A \\&\text{since } w = s^{-1}, u_1 = we \text{ and } u_2 = wr\end{aligned}$$

$$(x_1, y_1) = u_1 G + u_2 Q_A$$

$$\text{Hence } (x_1, y_1) = u_1 G + u_2 d_A G = kG$$

$$\text{because } Q_A = d_A G \text{ and } k = u_1 + u_2 d_A$$

We conclude that  $r = x_1 \pmod{n}$  by construction.

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## Data Encryption Standard, (call in 1973)

Lucifer designed in 1971 by Horst Feistel at IBM.

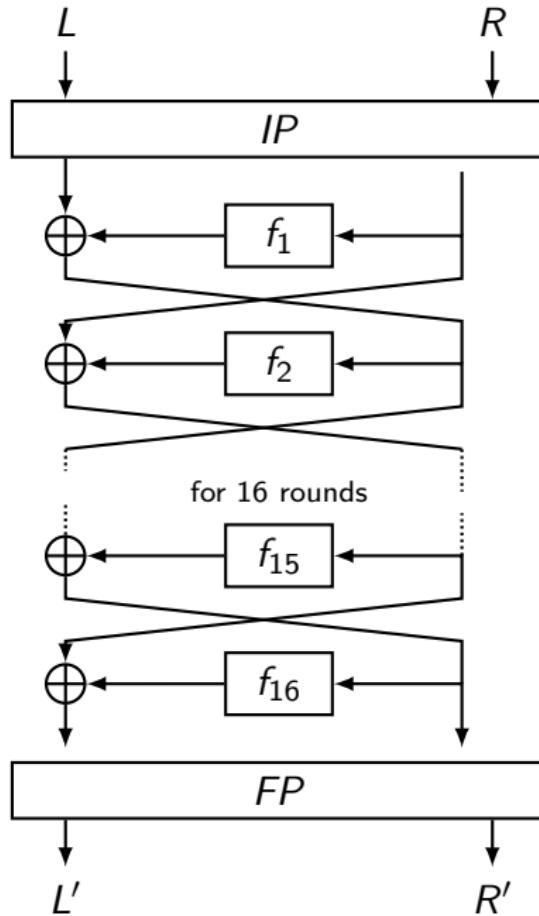
- ▶ Block cipher, encrypting 64-bit blocks
  - Uses 56 bit keys
  - Expressed as 64 bit numbers (8 bits parity checking)
- ▶ First cryptographic standard.
  - ▶ 1977 US federal standard (US Bureau of Standards)
  - ▶ 1981 ANSI private sector standard

## DES — overall form

- ▶ 16 rounds Feistel cipher + key-scheduler.
- ▶ Key scheduling algorithm derives subkeys  $K_i$  from original key  $K$ .
- ▶ Initial permutation at start, and inverse permutation at end.
- ▶  $f_i$  consists of two permutations and an s-box substitution.

$$L_{i+1} = R_i \text{ and } R_{i+1} = L_i \oplus f(R_i, K_i)$$

## DES — overall form



## DES — Subkey generation

First, produce two subkeys K1 and K2:

$$K1 = P8(LS1(P10(key)))$$

$$K2 = P8(LS2(LS1(P10(key))))$$

where P8, P10, LS1 and LS2 are bit substitution operators.

- ▶ P10 : 10 bits to 10 bits

|   |   |   |   |   |    |   |   |   |   |
|---|---|---|---|---|----|---|---|---|---|
| 3 | 5 | 2 | 7 | 4 | 10 | 1 | 9 | 8 | 6 |
|---|---|---|---|---|----|---|---|---|---|

- ▶ P8 : 10 bits to 8 bits

|   |   |   |   |   |   |    |   |
|---|---|---|---|---|---|----|---|
| 6 | 3 | 7 | 4 | 8 | 5 | 10 | 9 |
|---|---|---|---|---|---|----|---|

- ▶ LS1 ("left shift 1 bit" on 5 bit words) : 10 bits to 10 bits

|   |   |   |   |   |   |   |   |    |   |
|---|---|---|---|---|---|---|---|----|---|
| 2 | 3 | 4 | 5 | 1 | 7 | 8 | 9 | 10 | 6 |
|---|---|---|---|---|---|---|---|----|---|

- ▶ LS2 ("left shift 2 bit" on 5 bit words) : 10 bits to 10 bits

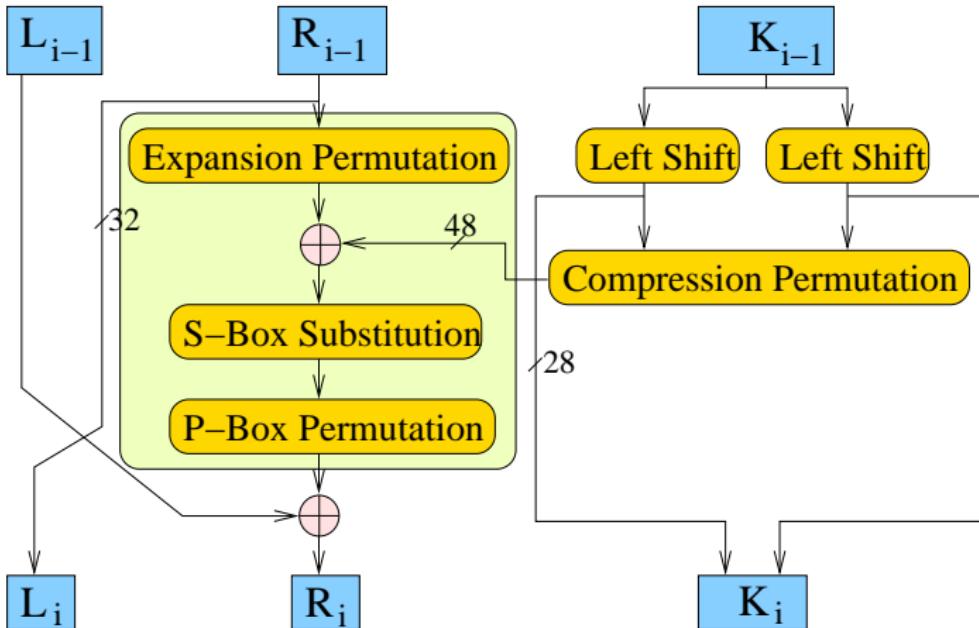
|   |   |   |   |   |   |   |    |   |   |
|---|---|---|---|---|---|---|----|---|---|
| 3 | 4 | 5 | 1 | 2 | 8 | 9 | 10 | 6 | 7 |
|---|---|---|---|---|---|---|----|---|---|

## DES — Before round subkey

Each half of the key schedule state is rotated left by a number of places.

| # Rds | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| Left  | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2  | 2  | 2  | 2  | 2  | 2  | 1  |

## DES — 1 round



$(b_1 b_6, b_2 b_3 b_4 b_5)$ ,  $C_j$  represents the binary value in the row  $b_1 b_6$  and column  $b_2 b_3 b_4 b_5$  of the  $S_j$  box.

## S-Boxes: S1, S2, S3, S4

|    |    |    |   |    |    |    |    |    |    |    |    |    |    |   |    |
|----|----|----|---|----|----|----|----|----|----|----|----|----|----|---|----|
| 14 | 4  | 13 | 1 | 2  | 15 | 11 | 8  | 3  | 10 | 6  | 12 | 5  | 9  | 0 | 7  |
| 0  | 15 | 7  | 4 | 14 | 2  | 13 | 1  | 10 | 6  | 12 | 11 | 9  | 5  | 3 | 8  |
| 4  | 1  | 14 | 8 | 13 | 6  | 2  | 11 | 15 | 12 | 9  | 7  | 3  | 10 | 5 | 0  |
| 15 | 12 | 8  | 2 | 4  | 9  | 1  | 7  | 5  | 11 | 3  | 14 | 10 | 0  | 6 | 13 |

|    |    |    |    |    |    |    |    |    |   |    |    |    |   |    |    |
|----|----|----|----|----|----|----|----|----|---|----|----|----|---|----|----|
| 15 | 1  | 8  | 14 | 6  | 11 | 3  | 4  | 9  | 7 | 2  | 13 | 12 | 0 | 5  | 10 |
| 3  | 13 | 4  | 7  | 15 | 2  | 8  | 14 | 12 | 0 | 1  | 10 | 6  | 9 | 11 | 5  |
| 0  | 14 | 7  | 11 | 10 | 4  | 13 | 1  | 5  | 8 | 12 | 6  | 9  | 3 | 2  | 15 |
| 13 | 8  | 10 | 1  | 3  | 15 | 4  | 2  | 11 | 6 | 7  | 12 | 0  | 5 | 14 | 9  |

|    |    |    |    |   |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|---|----|----|----|----|----|----|----|----|----|----|----|
| 10 | 0  | 9  | 14 | 6 | 3  | 15 | 5  | 1  | 13 | 12 | 7  | 11 | 4  | 2  | 8  |
| 13 | 7  | 0  | 9  | 3 | 4  | 6  | 10 | 2  | 8  | 5  | 14 | 12 | 11 | 15 | 1  |
| 13 | 6  | 4  | 9  | 8 | 15 | 3  | 0  | 11 | 1  | 2  | 12 | 5  | 10 | 14 | 7  |
| 1  | 10 | 13 | 0  | 6 | 9  | 8  | 7  | 4  | 15 | 14 | 3  | 11 | 5  | 2  | 12 |

|    |    |    |   |    |    |    |    |    |   |   |    |    |    |    |    |
|----|----|----|---|----|----|----|----|----|---|---|----|----|----|----|----|
| 7  | 13 | 14 | 3 | 0  | 6  | 9  | 10 | 1  | 2 | 8 | 5  | 11 | 12 | 4  | 15 |
| 13 | 8  | 11 | 5 | 6  | 15 | 0  | 3  | 4  | 7 | 2 | 12 | 1  | 10 | 14 | 9  |
| 10 | 6  | 9  | 0 | 12 | 11 | 7  | 13 | 15 | 1 | 3 | 14 | 5  | 2  | 8  | 4  |
| 3  | 15 | 0  | 6 | 10 | 1  | 13 | 8  | 9  | 4 | 5 | 11 | 12 | 7  | 2  | 14 |

## S-Boxes: S5, S6, S7 and S8

|    |    |    |    |    |    |    |    |    |    |    |    |    |   |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|---|----|----|
| 2  | 12 | 4  | 1  | 7  | 10 | 11 | 6  | 8  | 5  | 3  | 15 | 13 | 0 | 14 | 9  |
| 14 | 11 | 2  | 12 | 4  | 7  | 13 | 1  | 5  | 0  | 15 | 10 | 3  | 9 | 8  | 6  |
| 4  | 2  | 1  | 11 | 10 | 13 | 7  | 8  | 15 | 9  | 12 | 5  | 6  | 3 | 0  | 14 |
| 11 | 8  | 12 | 7  | 1  | 14 | 2  | 13 | 6  | 15 | 0  | 9  | 10 | 4 | 5  | 3  |

|    |    |    |    |   |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|---|----|----|----|----|----|----|----|----|----|----|----|
| 12 | 1  | 10 | 15 | 9 | 2  | 6  | 8  | 0  | 13 | 3  | 4  | 14 | 7  | 5  | 11 |
| 10 | 15 | 4  | 2  | 7 | 12 | 9  | 5  | 6  | 1  | 13 | 14 | 0  | 11 | 3  | 8  |
| 9  | 14 | 15 | 5  | 2 | 8  | 12 | 3  | 7  | 0  | 4  | 10 | 1  | 13 | 11 | 6  |
| 4  | 3  | 2  | 12 | 9 | 5  | 15 | 10 | 11 | 14 | 1  | 7  | 6  | 0  | 8  | 13 |

|    |    |    |    |    |   |    |    |    |    |   |    |    |    |   |    |
|----|----|----|----|----|---|----|----|----|----|---|----|----|----|---|----|
| 4  | 11 | 2  | 14 | 15 | 0 | 8  | 13 | 3  | 12 | 9 | 7  | 5  | 10 | 6 | 1  |
| 13 | 0  | 11 | 7  | 4  | 9 | 1  | 10 | 14 | 3  | 5 | 12 | 2  | 15 | 8 | 6  |
| 1  | 4  | 11 | 13 | 12 | 3 | 7  | 14 | 10 | 15 | 6 | 8  | 0  | 5  | 9 | 2  |
| 6  | 11 | 13 | 8  | 1  | 4 | 10 | 7  | 9  | 5  | 0 | 15 | 14 | 2  | 3 | 12 |

|    |    |    |   |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|---|----|----|----|----|----|----|----|----|----|----|----|----|
| 13 | 2  | 8  | 4 | 6  | 15 | 11 | 1  | 10 | 9  | 3  | 14 | 5  | 0  | 12 | 7  |
| 1  | 15 | 13 | 8 | 10 | 3  | 7  | 4  | 12 | 5  | 6  | 11 | 0  | 14 | 9  | 2  |
| 7  | 11 | 4  | 1 | 9  | 12 | 14 | 2  | 0  | 6  | 10 | 13 | 15 | 3  | 5  | 8  |
| 2  | 1  | 14 | 7 | 4  | 10 | 8  | 13 | 15 | 12 | 9  | 0  | 3  | 5  | 6  | 11 |

## Permutation P

|    |    |    |    |    |    |    |   |
|----|----|----|----|----|----|----|---|
| 58 | 50 | 42 | 34 | 26 | 18 | 10 | 2 |
| 60 | 52 | 44 | 36 | 28 | 20 | 12 | 4 |
| 62 | 54 | 46 | 38 | 30 | 22 | 14 | 6 |
| 64 | 56 | 48 | 40 | 32 | 24 | 16 | 8 |
| 57 | 49 | 41 | 33 | 25 | 17 | 9  | 1 |
| 59 | 51 | 43 | 35 | 27 | 19 | 11 | 3 |
| 61 | 53 | 45 | 37 | 29 | 21 | 13 | 5 |
| 63 | 55 | 47 | 39 | 31 | 23 | 15 | 7 |

## Decryption DES

Use inverse sequence key.

- ▶  $IP(C) = IP(IP^{-1}(R_{16} || L_{16}))$
- ▶  $L'_0 = R_{16}$  and  $R'_0 = L_{16}$

$$L'_1 = R'_0 = L_{16} = R_{15}$$

$$R'_1 = L'_0 \oplus f(R'_0, K'_0)$$

$$R'_1 = R_{16} \oplus f(L_{16}, K_{15})$$

$$R'_1 = R_{16} \oplus f(R_{15}, K_{15})$$

$$R'_1 = L_{15}$$

Recall  $L_{i+1} = R_i$  and  $R_{i+1} = L_i \oplus f(R_i, K_i)$

## Property of DES

DES exhibits the complementation property, namely that

$$E_K(P) = C \Leftrightarrow E_{\bar{K}}(\bar{P}) = \bar{C}$$

where  $\bar{x}$  is the bitwise complement of  $x$ .  $E_K$  denotes encryption with key  $K$ . Then  $P$  and  $C$  denote plaintext and ciphertext blocks respectively.

## Anomalies of DES

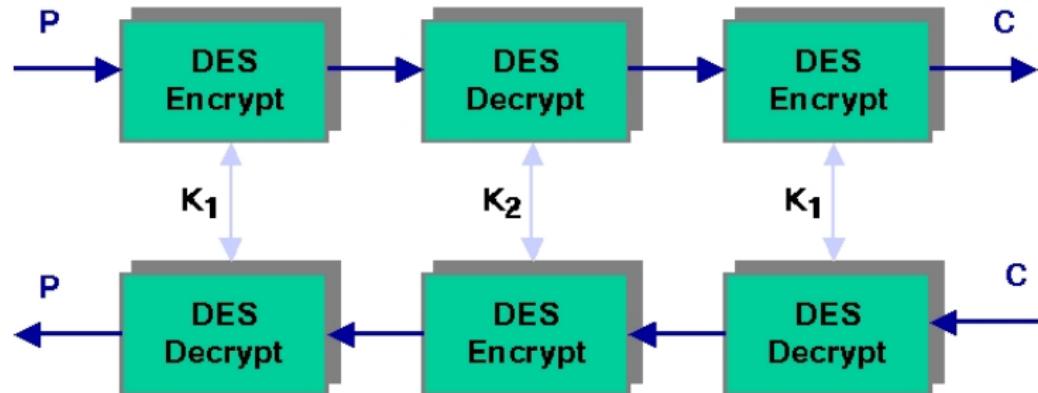
- ▶ Existence of 6 pairs of *semi-weak keys*:  $E_{k_1}(E_{k_2}(x)) = x$ .
  - ▶ 0x011F011F010E010E and 0x1F011F010E010E01
  - ▶ 0x01E001E001F101F1 and 0xE001E001F101F101
  - ▶ 0x01FE01FE01FE01FE and 0xFE01FE01FE01FE01
  - ▶ 0x1FE01FE00EF10EF1 and 0xE01FE01FF10EF10E
  - ▶ 0xFFE1FFE0EFE0EFE and 0xFE1FFE1FFE0EFE0E
  - ▶ 0xE0FEE0FEF1FEF1FE and 0xFEE0FEE0FEF1FEF1

# Security of DES

- ▶ No security proofs or reductions known
- ▶ Main attack: exhaustive search
  - ▶ 7 hours with 1 million dollar computer (in 1993).
  - ▶ 7 days with \$10,000 FPGA-based machine (in 2006).
- ▶ Mathematical attacks
  - ▶ Not know yet.
  - ▶ But it is possible to reduce key space from  $2^{56}$  to  $2^{43}$  using (linear) cryptanalysis.
    - ▶ To break the full 16 rounds, differential cryptanalysis requires  $2^{47}$  chosen plaintexts (Eli Biham and Adi Shamir).
    - ▶ Linear cryptanalysis needs  $2^{43}$  known plaintexts (Matsui, 1993)

## Triple DES

- ▶ Use three stages of encryption instead of two.



- ▶ Compatibility is maintained with standard DES ( $K_2 = K_1$ ).
- ▶ No known practical attack  
⇒ brute-force search with  $2^{112}$  operations.

## Advanced Encryption Standard

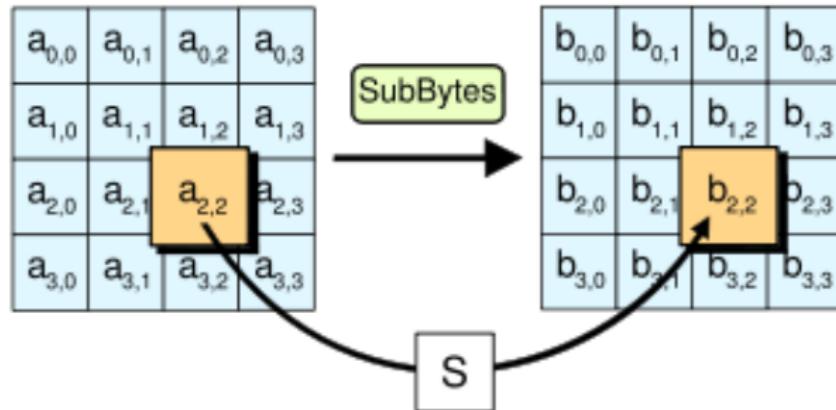
- ▶ Block cipher, approved for use by US Government in 2002.  
Very popular standard, designed by two Belgian cryptographers.
- ▶ Block-size = 128 bits, Key size = 128, 192, or 256 bits.
- ▶ Uses various substitutions and transpositions + key scheduling, in different rounds.
- ▶ Algorithm believed secure. Only attacks are based on side channel analysis, i.e., attacking implementations that inadvertently leak information about the key.

| Key Size | Round Number |
|----------|--------------|
| 128      | 10           |
| 192      | 12           |
| 256      | 14           |

## AES: High-level cipher algorithm

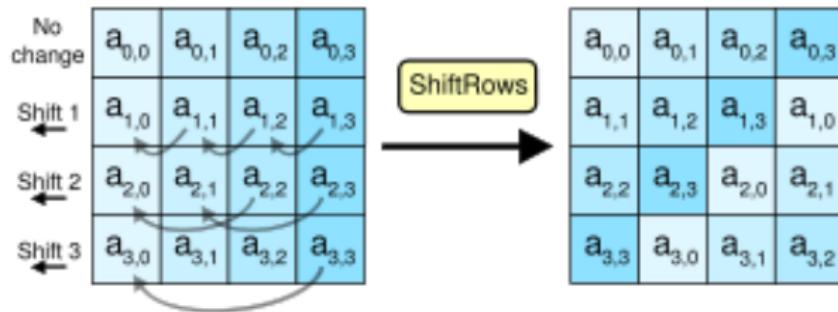
- ▶ KeyExpansion using Rijndael's key schedule
- ▶ Initial Round: AddRoundKey
- ▶ Rounds:
  1. SubBytes: a non-linear substitution step where each byte is replaced with another according to a lookup table.
  2. ShiftRows: a transposition step where each row of the state is shifted cyclically a certain number of steps.
  3. MixColumns: a mixing operation which operates on the columns of the state, combining the four bytes in each column
  4. AddRoundKey: each byte of the state is combined with the round key; each round key is derived from the cipher key using a key schedule.
- ▶ Final Round (no MixColumns)
  1. SubBytes
  2. ShiftRows
  3. AddRoundKey

## AES: SubBytes



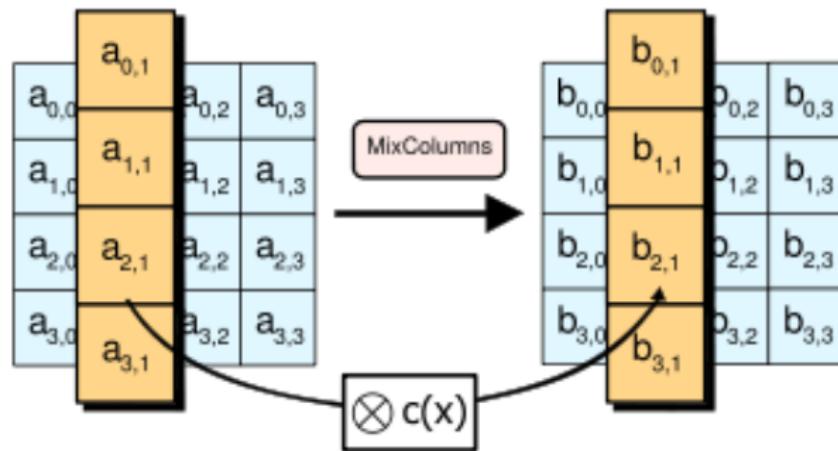
SubBytes: a non-linear substitution step where each byte is replaced with another according to a lookup table.

## AES: ShiftRows



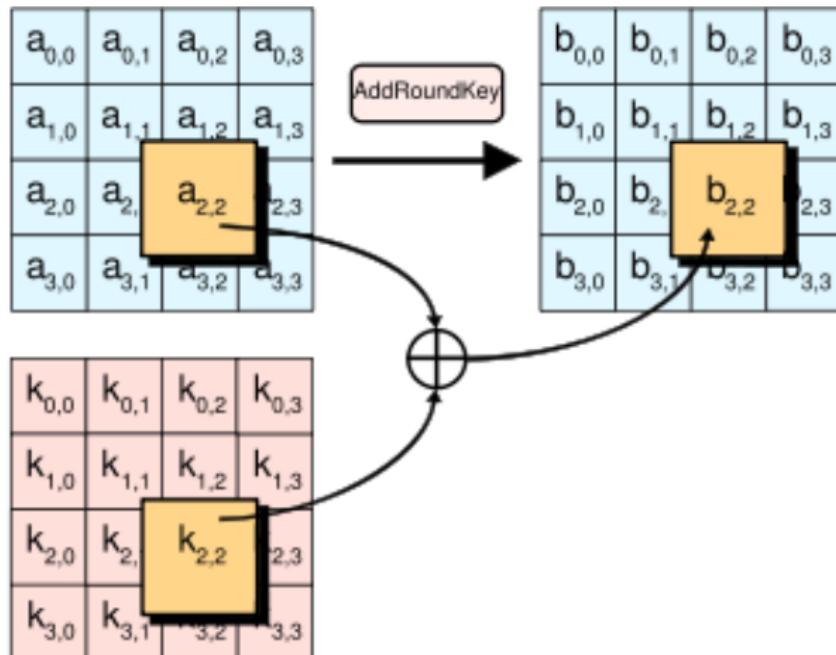
ShiftRows: a transposition step where each row of the state is shifted cyclically a certain number of steps.

## AES: MixColumns



MixColumns: a mixing operation which operates on the columns of the state, combining the four bytes in each column

## AES: AddRoundKey



AddRoundKey: each byte of the state is combined with the round key; each round key is derived from the cipher key using a key schedule.

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## Electronic Book Code (ECB)

Each block of the same length is encrypted separately using the same key  $K$ . In this mode, only the block in which the flipped bit is contained is changed. Other blocks are not affected.

# ECB Encryption Algorithm

**algorithm**  $E_K(M)$

if ( $|M| \bmod n \neq 0$  or  $|M| = 0$ ) then return FAIL

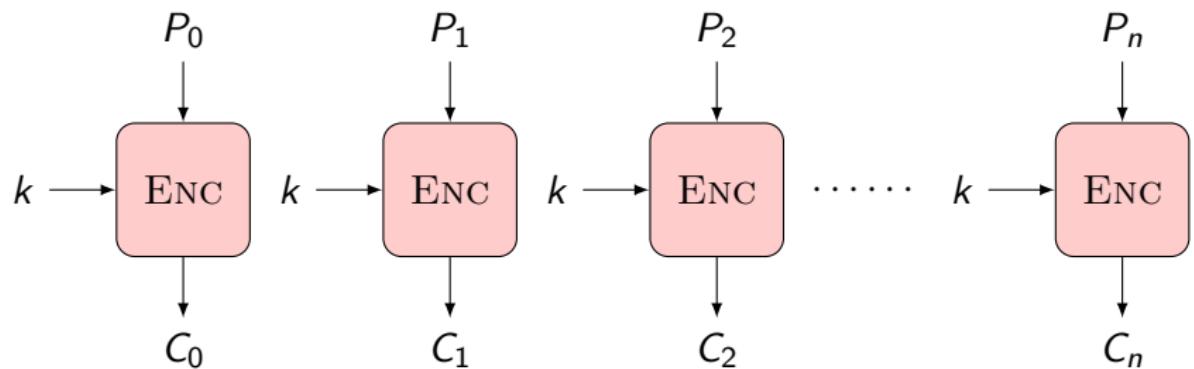
Break  $M$  into  $n$ -bit blocks  $M[1] \dots M[m]$

for  $i = 1$  to  $m$  do  $C[i] = E_K(M[i])$

$C = C[1] \dots C[m]$

return  $C$

# ECB Encryption



# ECB Decryption Algorithm

**algorithm**  $D_K(C)$

if ( $|C| \bmod n \neq 0$  or  $|C| = 0$ ) then return FAIL

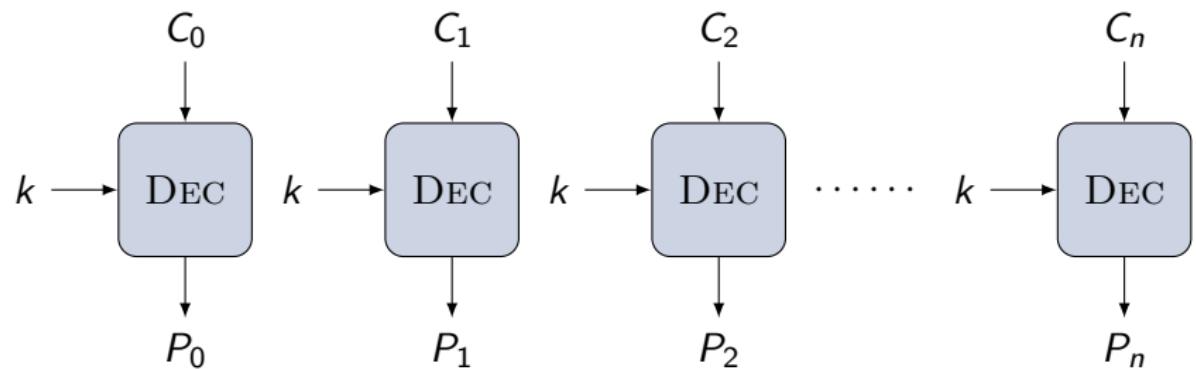
Break  $C$  into  $n$ -bit blocks  $C[1] \dots C[m]$

for  $i = 1$  to  $m$  do  $M[i] = D_K(C[i])$

$M = M[1] \dots M[m]$

return  $M$

## ECB Decryption



## Cipher-block chaining (CBC)

If the first block has index 1, the mathematical formula for CBC encryption is

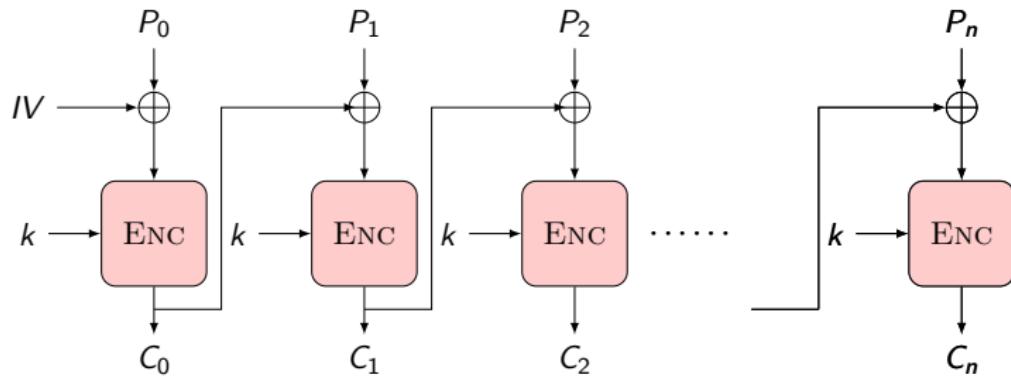
$$C_i = E_K(P_i \oplus C_{i-1}), C_0 = IV$$

while the mathematical formula for CBC decryption is

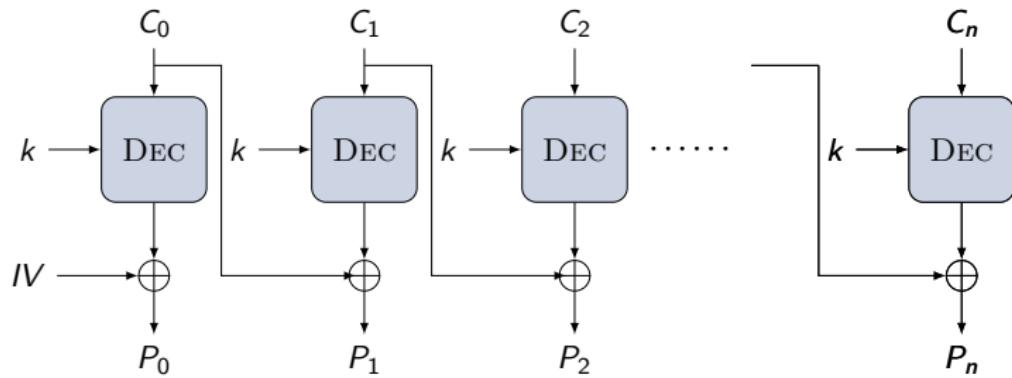
$$P_i = D_K(C_i) \oplus C_{i-1}, C_0 = IV$$

CBC has been the most commonly used mode of operation.

## CBC Encryption



## CBC Decryption



# The cipher feedback (CFB)

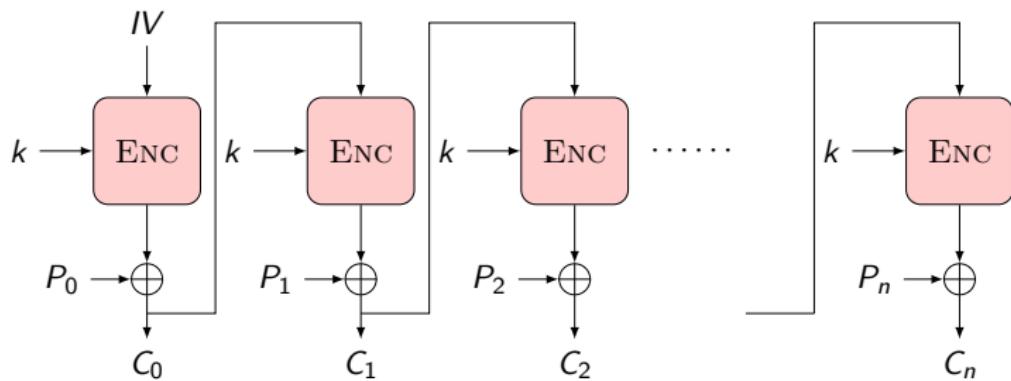
A close relative of CBC:

$$C_i = E_K(C_{i-1}) \oplus P_i$$

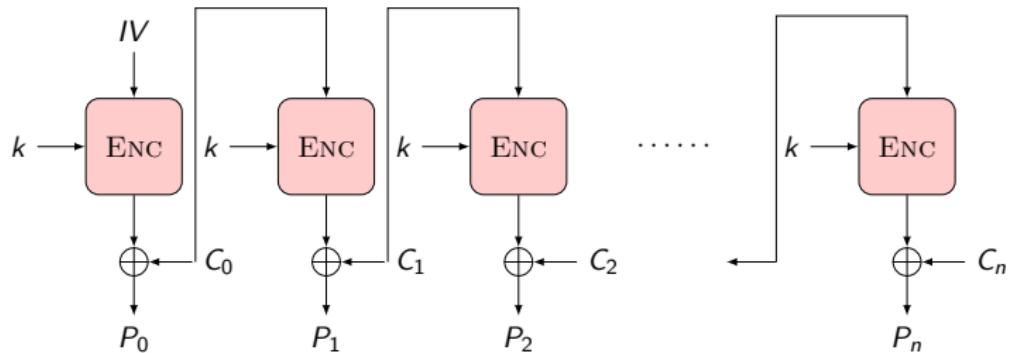
$$P_i = E_K(C_{i-1}) \oplus C_i$$

$$C_0 = \text{IV}$$

# CFB Encryption



# CFB Decryption



## Output feedback (OFB)

Because of the symmetry of the XOR operation, encryption and decryption are exactly the same:

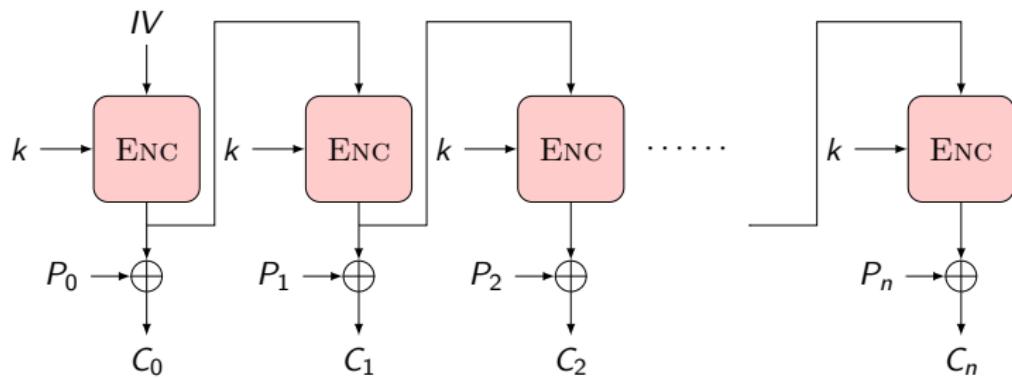
$$C_i = P_i \oplus O_i$$

$$P_i = C_i \oplus O_i$$

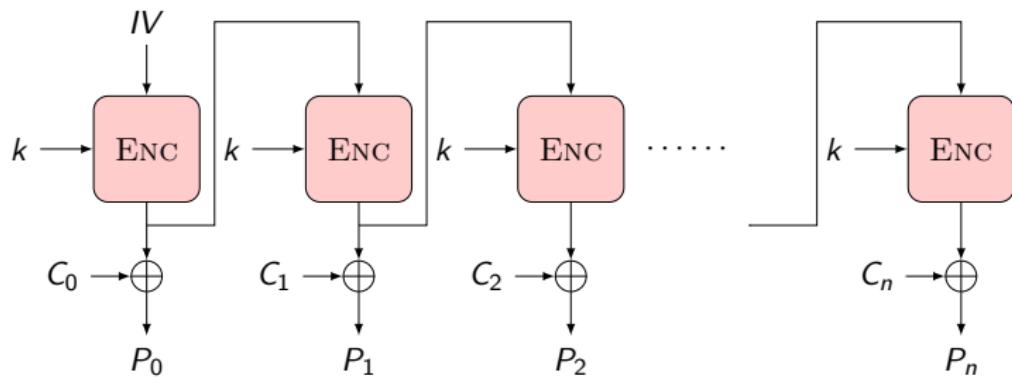
$$O_i = E_K(O_{i-1})$$

$$O_0 = \text{IV}$$

# OFB encryption



# OFB Decryption



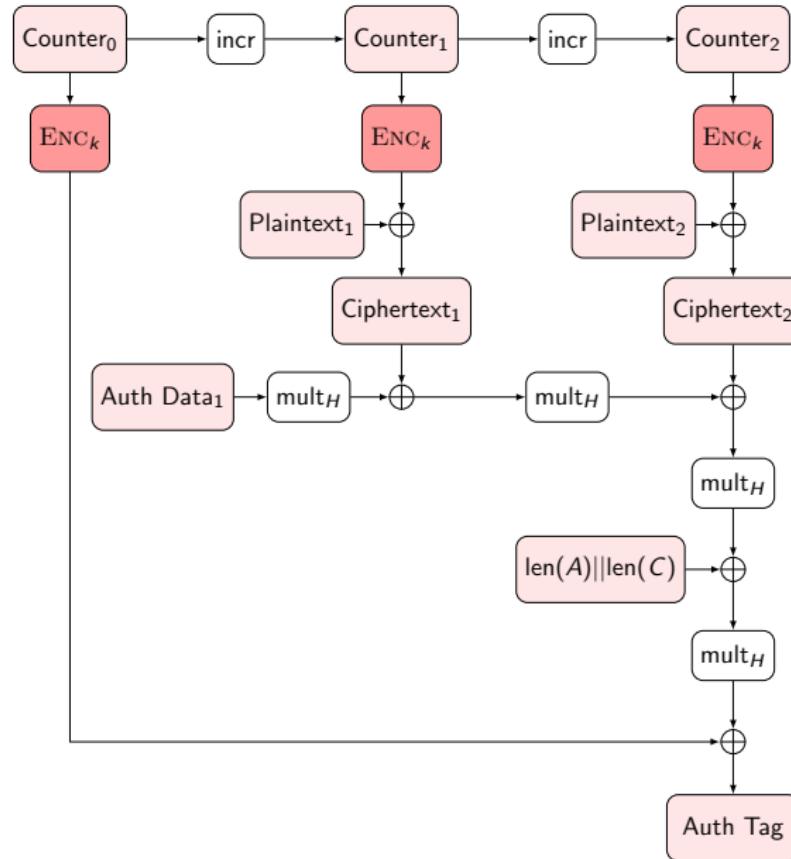
## Counter Mode (CTR)

$$C_0 = IV$$

$$C_i = P_i \oplus E_k(IV + i - 1)$$

$$P_i = C_i \oplus E_k(IV + i - 1)$$

# GCM Galois/Counter Mode by D. McGrew and J. Viega



## GCM

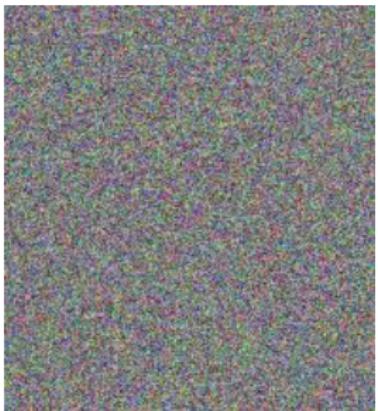
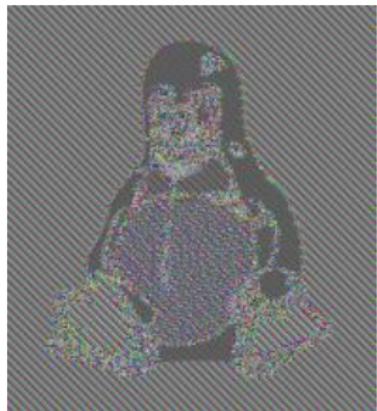
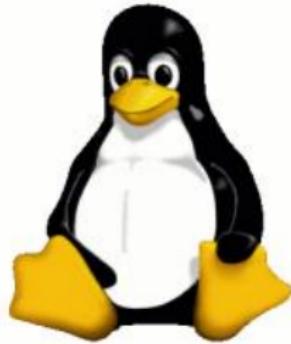
$GF(2^{128})$  est défini par  $x^{128} + x^7 + x^2 + x + 1$

$$S_i = \begin{cases} A_i & \text{for } i = 1, \dots, m-1 \\ A_m^* \parallel 0^{128-v} & \text{for } i = m \\ C_{i-m} & \text{for } i = m+1, \dots, m+n-1 \\ C_n^* \parallel 0^{128-u} & \text{for } i = m+n \\ \text{len}(A) \parallel \text{len}(C) & \text{for } i = m+n+1 \end{cases}$$

where  $\text{len}(A)$  and  $\text{len}(C)$  are the 64-bit representations of the bit lengths of  $A$  and  $C$ , respectively,  $v = \text{len}(A) \bmod 128$  is the bit length of the final block of  $A$ ,  $u = \text{len}(C) \bmod 128$  is the bit length of the final block of  $C$ .

$$X_i = \sum_{j=1}^i S_j \cdot H^{i-j+1} = \begin{cases} 0 & \text{for } i = 0 \\ (X_{i-1} \oplus S_i) \cdot H & \text{for } i = 1, \dots, m+n+1 \end{cases}$$

## ECB vs Others



# Outline

History of Cryptography

Classical Asymmetric Encryptions

Elliptic Curves

Classical Symetric Encryptions

DES

3-DES

AES

Modes

Hash Functions

Conclusion

# “Classifications” of Hash Functions

## Unkeyed Hash function

- ▶ Modification Code Detection (MDC)
- ▶ Data integrity
- ▶ Fingerprints of messages
- ▶ Other applications

## Keyed Hash function

- ▶ Message Authentication Code (MAC)
- ▶ Password Verification in unencrypted password-image files.
- ▶ Key confirmation or establishment
- ▶ Time-stamping
- ▶ Others applications

# Hash Functions

A hash function  $H$  takes as input a bit-string of any finite length and returns a corresponding 'digest' of **fixed length**.

$$h : \{0, 1\}^* \rightarrow \{0, 1\}^n$$

$$H(Alice) =$$



**Definition (Pre-image resistance (One-way) OWHF)**

Given an output  $y$ , it is computationally infeasible to compute  $x$  such that

$$h(x) = y$$

## Properties of hash functions

### 2nd Pre-image resistance (weak-collision resistant) CRHF

Given an input  $x$ , it is computationally infeasible to compute  $x'$  such that

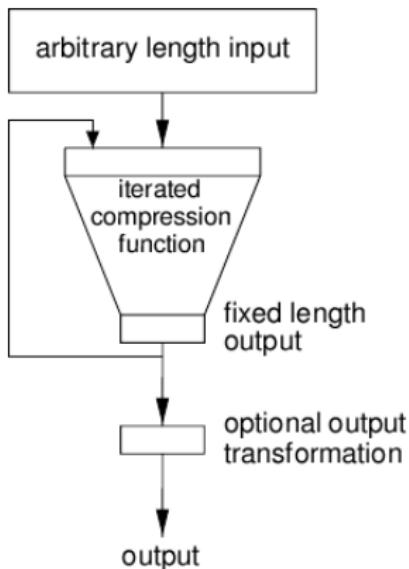
$$h(x') = h(x)$$

### Collision resistance (strong-collision resistant)

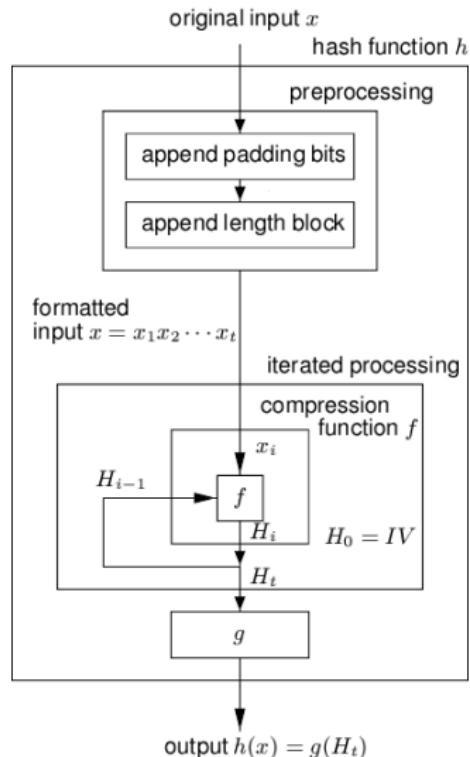
It is computationally infeasible to compute  $x$  and  $x'$  such that

$$h(x) = h(x')$$

# Basic construction of hash functions



# Basic construction of hash functions



## Basic construction of hash functions (Merkle-Damgård)

$$f : \{0,1\}^m \rightarrow \{0,1\}^n$$

1. Break the message  $x$  to hash in blocks of size  $m - n$ :

$$x = x_1 x_2 \dots x_t$$

2. Pad  $x_t$  with zeros as necessary.
3. Define  $x_{t+1}$  as the binary representation of the bit length of  $x$ .
4. Iterate over the blocks:

$$H_0 = 0^n$$

$$H_i = f(H_{i-1} || x_i)$$

$$h(x) = H_{t+1}$$

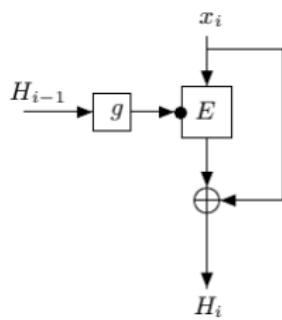
# Basic construction of hash functions

## Theorem

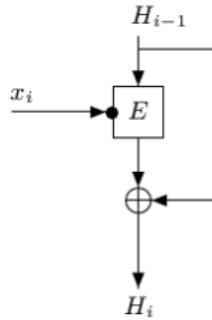
*If the compression function  $f$  is collision resistant, then the obtained hash function  $h$  is collision resistant.*

# Hash functions based on (MDC) block ciphers

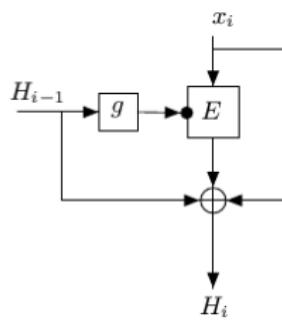
Matyas-Meyer-Oseas



Davies-Meyer

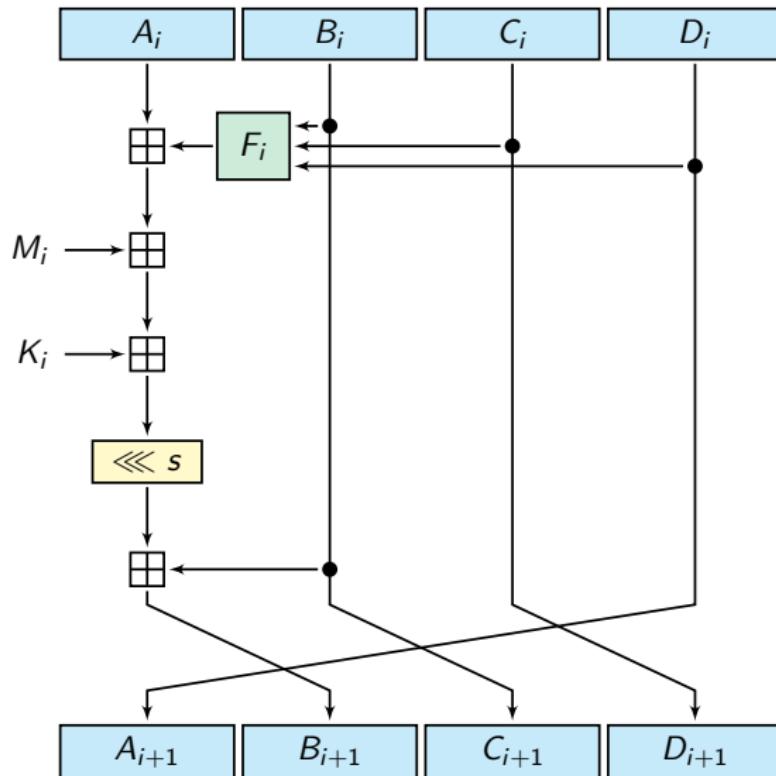


Miyaguchi-Preneel



# MD5 by Ron Rivest in 1991

For each 512-bit block of plaintext



$K_i$  denotes a 32-bit constant, different for each operation Addition 108 / 136

## MD5 by Ron Rivest in 1991

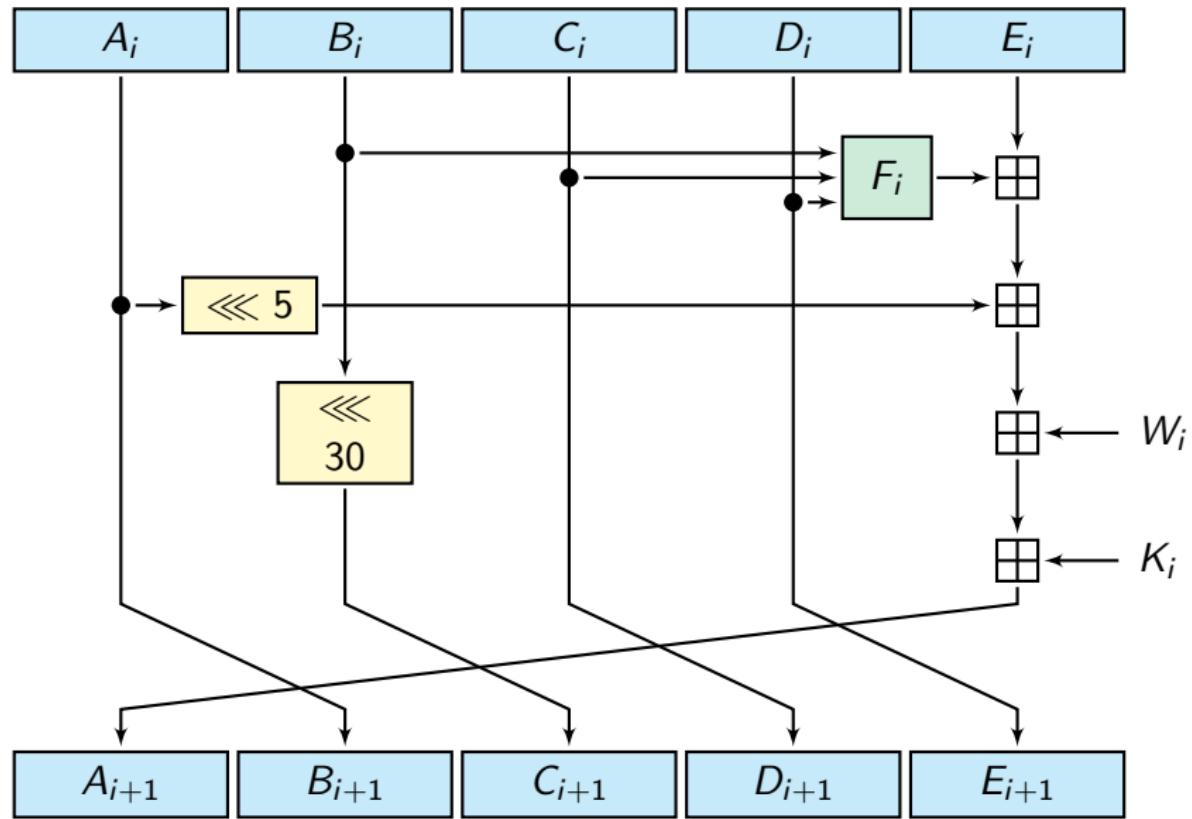
There are four possible functions F; a different one is used in each round:

- ▶  $F(B, C, D) = (B \wedge C) \vee (\neg B \wedge D)$
- ▶  $G(B, C, D) = (B \wedge D) \vee (C \wedge \neg D)$
- ▶  $H(B, C, D) = B \oplus C \oplus D$
- ▶  $I(B, C, D) = C \oplus (B \vee \neg D)$

## MD5 Cryptanalysis

- ▶ In 1993, Den Boer and Bosselaers gave a "pseudo-collision" two different initialization vectors of compression function which produce an identical digest.
- ▶ In 1996, Dobbertin announced a collision of the compression function of MD5.
- ▶ 17 August 2004, collisions for the full MD5 by Xiaoyun Wang, Dengguo Feng, Xuejia Lai, and Hongbo Yu.
- ▶ On 1 March 2005, Arjen Lenstra, Xiaoyun Wang, and Benne de Weger demonstrated construction of two X.509 certificates with different public keys and the same MD5 hash value.
- ▶ A few days later, Vlastimil Klima able to construct MD5 collisions in a few hours on a single notebook computer.
- ▶ On 18 March 2006, Klima published an algorithm that can find a collision within one minute on a single notebook computer, using a method he calls tunneling.
- ▶ On 24 December 2010, Tao Xie and Dengguo Feng announced the first published single-block (512 bit) MD5 collision.

# SHA-1



# Collision for PDF



$\text{SHA1}(\text{TP}/\text{SHA1}/\text{a.pdf}) =$

a5a678701d8b2ab07c96d101b3331fb4992f0980

$\text{SHA1}(\text{TP}/\text{SHA1}/\text{b.pdf}) =$

a5a678701d8b2ab07c96d101b3331fb4992f0980

# List of Hash Functions

| Algorithm   | Output size      | Internal state size | Block size | Length size | Word size | Collision  |
|-------------|------------------|---------------------|------------|-------------|-----------|------------|
| HAVAL       | 256/.../128      | 256                 | 1024       | 64          | 32        | Yes        |
| MD2         | 128              | 384                 | 128        | No          | 8         | Almost     |
| MD4         | 128              | 128                 | 512        | 64          | 32        | Yes        |
| MD5         | 128              | 128                 | 512        | 64          | 32        | Yes        |
| PANAMA      | 256              | 8736                | 256        | No          | 32        | Yes        |
| RadioGatún  | Arbitrarily long | 58 words            | 3 words    | No          | 1-64      | No         |
| RIPEMD      | 128              | 128                 | 512        | 64          | 32        | Yes        |
| RIPEMD      | 128/256          | 128/256             | 512        | 64          | 32        | No         |
| RIPEMD      | 160/320          | 160/320             | 512        | 64          | 32        | No         |
| SHA-0       | 160              | 160                 | 512        | 64          | 32        | Yes        |
| SHA-1       | 160              | 160                 | 512        | 64          | 32        | With flaws |
| SHA-256/224 | 256/224          | 256                 | 512        | 64          | 32        | No         |
| SHA-512/384 | 512/384          | 512                 | 1024       | 128         | 64        | No         |
| Tiger(2)    | 192/160/128      | 192                 | 512        | 64          | 64        | No         |
| WHIRLPOOL   | 512              | 512                 | 512        | 256         | 8         | No         |

## SHA-3 Zoo

64 Submissions, 54 selected,

1. \* BLAKE Jean-Philippe Aumasson
2. Blue Midnight Wish Svein Johan Knapskog
3. CubeHash Daniel J. Bernstein preimage
4. ECHO Henri Gilbert
5. Fugue Charanjit S. Jutla
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10. Luffa Dai Watanabe
11. Shabal Jean-François Misarsky
12. SHAvite-3 Orr Dunkelman
13. SIMD Gaëtan Leurent
14. \* Skein Bruce Schneier

## SHA-3 = Keccak (sponge + compression)

### Authors

- ▶ Guido Bertoni (Italy) of STMicroelectronics,
- ▶ Joan Daemen (Belgium) of STMicroelectronics,
- ▶ Michaël Peeters (Belgium) of NXP Semiconductors, and
- ▶ Gilles Van Assche (Belgium) of STMicroelectronics.

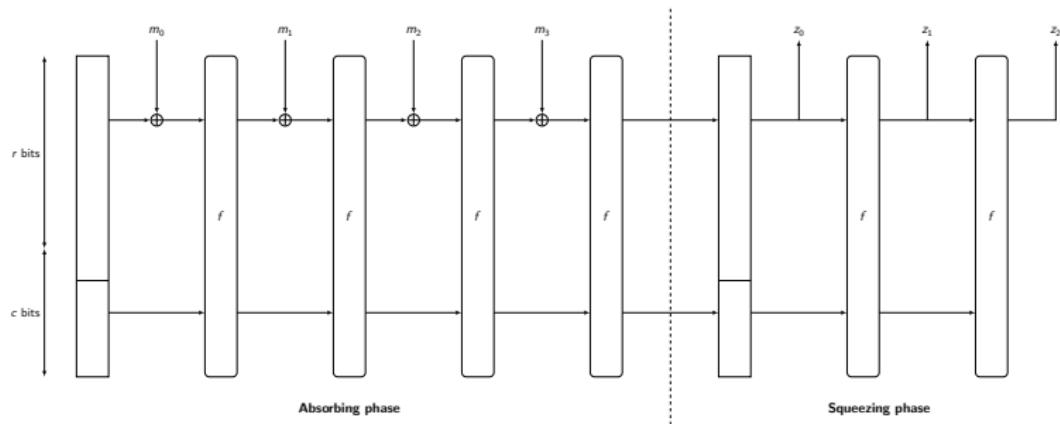
## SHA-3 = Keccak

$$h : \{1, 0\}^* \rightarrow \{1, 0\}^n$$

- ▶ MD5:  $n = 128$  (Ron Rivest, 1992)
- ▶ SHA-1:  $n = 160$  (NSA, NIST, 1995)
- ▶ SHA-2:  $n \in \{224, 256, 384, 512\}$  (NSA, NIST, 2001)
- ▶ SHA-3:  $n$  is arbitrary (NSA, NIST, 2012)

# SHA-3 = Keccak is a sponge based hash

$$H(P_0|P_1|\dots|P_i) = Z_0|Z_1|\dots|Z_I$$



$$b = r + c$$

- ▶ r bits of rate
- ▶ c bits of capacity (security parameter)

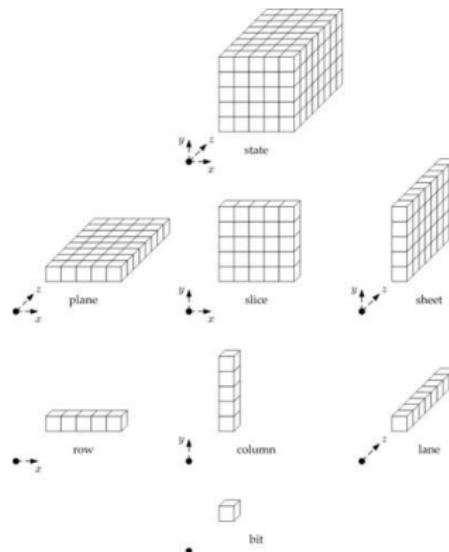
## Inside Keccak

- ▶ 7 permutations:  $b \in \{25, 50, 100, 200, 400, 800, 1600\}$
- ▶ ... from toy over lightweight to high-speed ...
- ▶ SHA-3 instance:  $r = 1088$  and  $c = 512$ 
  - ▶ permutation width: 1600
  - ▶ security strength 256: post-quantum sufficient
- ▶ Lightweight instance:  $r = 40$  and  $c = 160$ 
  - ▶ permutation width: 200
  - ▶ security strength 80: same as (initially expected from) SHA-1

## SHA-3 = Keccak f Setting

Defined for word of size,  $w = 2^l$  bits (if  $l = 6$  64-bit words )

State is  $5 \times 5 \times w$  array of bits ( $a[i][j][k]$ )



- ▶ state =  $5 \times 5$  lanes , each containing  $2^l$  bits
- ▶ (  $5 \times 5$ )-bit slices,  $2^l$  of them

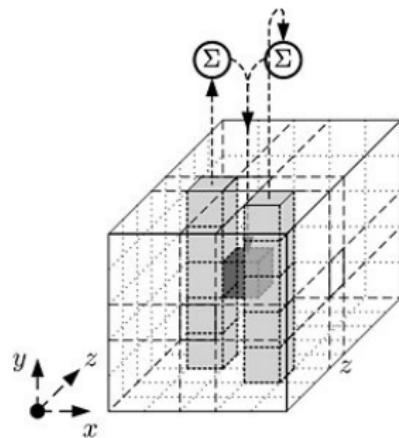
## SHA-3 = Keccak

The basic block permutation function consists of  $12 + 2 \times l$  iterations of following sub-rounds.

1. step  $\Theta$
2. step  $\rho$
3. step  $\pi$
4. step  $\chi$
5. step  $\iota$

## Keccak $\Theta$

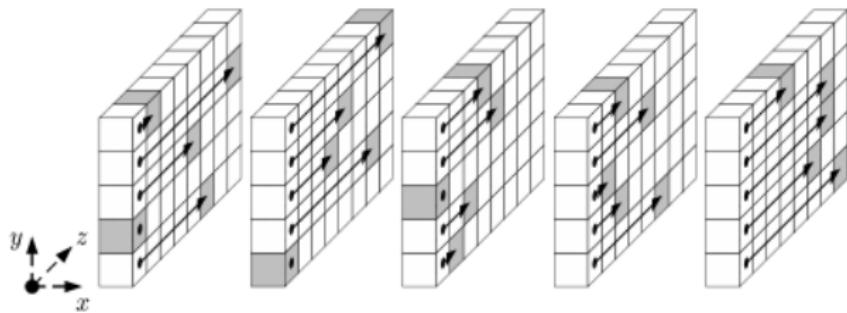
1. Compute the parity of each of the 5-bit columns
2.  $\oplus$  the sum of  $a[x-1][y][z]$  and of  $a[x+1][y][z-1]$  into  $a[x][y][z]$ .



$$a[i][j][k] \oplus = \text{parity}(a[0..4][j - 1][k]) \oplus \text{parity}(a[0..4][j + 1][k - 1])$$

## Keccak $\rho$

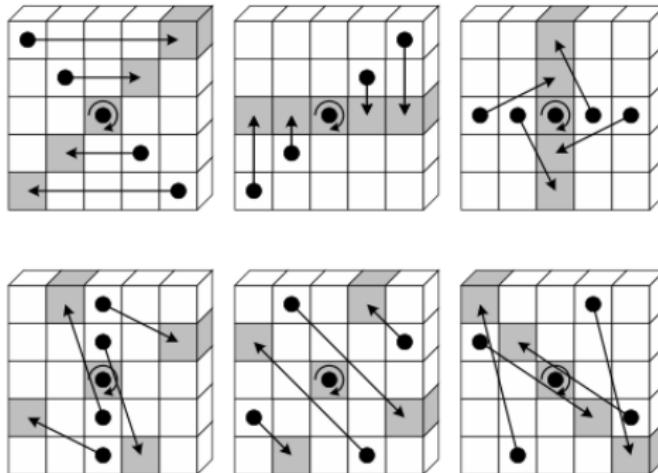
Bitwise rotate each of the 25 words by a different rotation.



$a[0][0]$  is not rotated, and for all  $0 \leq t < 24$   
 $a[i][j][k] = a[i][j][k - (t + 1)(t + 2)/2]$ , where  
$$\begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}^t \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

# Keccak $\pi$

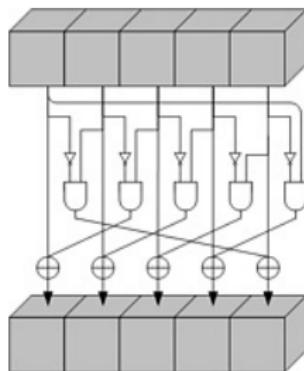
Permute the 25 words in a fixed pattern.



$$a[i][j] = a[j][2i + 3j]$$

## Keccak $\chi$

Bitwise combine along rows, using  $a = a \oplus (\neg b \& c)$ .



$$a[i][j][k] \oplus = \neg a[i][j+1][k] \& a[i][j+2][k]$$

This is the only non-linear operation in SHA-3.

## Keccak $\iota$

Exclusive-or a round constant into one word of the state.

- ▶ In round  $n$ , for  $0 \leq m \leq l$ ,  $a[0][0][2m - 1]$  is exclusive-ORed with bit  $m + 7n$  of a degree-8 LFSR (Linear Feedback Shift Register) sequence.

This breaks the symmetry that is preserved by the other sub-rounds.

## Hash Functions

A hash function  $\mathcal{H}$  takes as input a bit-string and returns a corresponding 'digest' of fixed length. Good hash functions are :

- ▶ collision-free:  $\mathcal{H}(x) = \mathcal{H}(y) \Rightarrow x = y$
- ▶ non-malleable:  $x \mathcal{R} y \not\Rightarrow \mathcal{H}(x) \mathcal{R} \mathcal{H}(y)$

$$\mathcal{H}(Alice) \neq \mathcal{H}(Bob)$$

# Properties of hash functions

## Definition (Preimage resistance)

Given an output  $y$ , it is computationally infeasible to compute  $x$  such that

$$h(x) = y$$

## Definition (2nd Preimage resistance)

Given an input  $x$ , it is computationally infeasible to compute  $x'$  such that

$$h(x') = h(x)$$

## Properties of hash functions

### Definition (Collision resistance)

It is computationally infeasible to compute  $x$  and  $x'$  such that

$$h(x) = h(x')$$

# Properties of hash functions

Alternate terminology:

- ▶ *pre-image resistant*  $\equiv$  *one-way*
- ▶ *2nd pre-image resistant*  $\equiv$  *weak collision resistant*
- ▶ *collision resistant*  $\equiv$  *strong collision resistant*

## Use of hash functions

Idea: compute a condensed message  $y$  from a given message  $m$ .  
The condensed should be specific to the message.

- ▶ Use  $y$  in place of  $m$  in a trustworthy way.
- ▶ "*Did you get  $m$  correctly? Here's  $y$  to check.*" (file-sharing)
- ▶ "*Could you decrypt  $c$  correctly?*"
- ▶ "*I sign  $y$  to prove that I wrote  $m$ .*"

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10. **Luffa** by Dai Watanabe
11. **Shabal** by Jean-François Misarsky
12. **SHAvite-3** by Orr Dunkelman
13. **SIMD** by Gaëtan Leurent
14. \* **Skein** by Bruce Schneier

## DMAC (CBC-MAC variant)

### Example

```
 $c_1 := m_1;$ 
for  $i = 2$  to  $n$  do:
     $z_i := c_{i-1} \oplus m_i$ 
     $c_i := E(z_i);$ 
tag :=  $E'(c_n);$ 
```

# HMAC

## Example

```
 $z_1 := k \| m_1;$ 
 $c_1 := \mathcal{H}(z_1);$ 
for  $i = 2$  to  $n$  do:;
 $z_i := c_{i-1} \| m_i$ 
 $c_i := \mathcal{H}(z_i)$ 
 $z' := k' || c_n;$ 
tag :=  $\mathcal{H}(z');$ 
```

# Outline

History of Cryptography

Classical Asymmetric Encryptions

Elliptic Curves

Classical Symetric Encryptions

DES

3-DES

AES

Modes

Hash Functions

Conclusion

## 5 Things to Bring Home

1. Several **security challenges** in IoT
2. Security has to be taken **at the design** of IoT
3. Designing secure protocols is **difficult**
4. **Tradeoff** between security, battery, CPU and price.
5. Use the **adapted cryptographic primitives**.



*Protocol + Properties + Intruder ⇒ Security*