Security and Cryptography just by images

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Secrecy or Confidentiality

Alice communicates with the White rabbit via a network.



Secrecy or Confidentiality

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Authentication



"On the Internet, nobody knows you're a dog."

Mechanisms for Authentication

5 / 63



1

Mechanisms for Authentication

5 / 63



1.



Mechanisms for Authentication









Mechanisms for Authentication







3.







Mechanisms for Authentication



Strong authentication combination of factors.







4.

1



Other security properties

- Integrity: No improper modification of information
- ► Availability: No improper impairment of functionality/service
- Non-repudiation (also called accountability) is where one can establish responsibility for actions.
- Privacy or Anonymity: secrecy of principal identities or communication relationships.
- ▶ etc ...

Symmetric key and public key encryption

• Symmetric key encryption



• Public key encryption



Outline

Motivations

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Two Examples

Outline

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Two Examples

History of Cryptography

Outline

Motivations

Two Examples

- History of Cryptography
- Cryptographic Security Intuitions

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Secret Sharing

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Secret Sharing

Conclusion

Security and Cryptography just by images Two Examples

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Motivations

Two Examples

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- Secret Sharing
- Conclusion

Security and Cryptography just by images Two Examples

Symetric Encryption for GSM communication



SIM card contains a shared secret key used for authenticating phones and operators, then creating key session for communication.

- 1. Message is encrypted and sent by Alice.
- 2. The antenna receives the message then uncrypted.
- 3. Message is encrypted by the antenna with the second key.
- 4. Second mobile uncrypted the communication.

Hash Functions

A hash function H takes as input a bit-string of any finite length and returns a corresponding 'digest' of fixed length.



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Hash function, e.g. Software Installation



Integrity of the downloaded file.

- 1. Download on server 1 the software.
- 2. Download on server 2 the hash of the software.
- 3. Check the integrity of the software.

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions Logical Attacks Interactive Zero Knowledge Proofs

Secret Sharing

Conclusion

Information hiding



- Cryptology: the study of secret writing.
- Steganography: the science of hiding messages in other messages.
- Cryptography: the science of secret writing.
 Note: terms like encrypt, encode, and encipher are often (loosely and wrongly) used interchangeably

Slave



Historical ciphers

Used 4000 years ago by Egyptians to encipher hieroglyphics.

- ▶ 2000 years ago Julius Caesar used a simple substitution cipher.
- Leon Alberti devised a cipher wheel, and described the principles of frequency analysis in the 1460s.

Substitution cipher examples

► L oryh brx

► L oryh brx = I LOVE YOU

Caesar cipher: each plaintext character is replaced by the character three to the right modulo 26.

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 ROT13: shift each letter by 13 places.
 Under Unix: tr a-zA-Z n-za-mN-ZA-M.
- ▶ 2-25-5 2-25-5

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Caesar cipher: each plaintext character is replaced by the character three to the right modulo 26.

- Zngurzngvdhrf = Mathematiques
 ROT13: shift each letter by 13 places.
 Under Unix: tr a-zA-Z n-za-mN-ZA-M.
- 2-25-5 2-25-5 = BYE BYE
 Alphanumeric: substitute numbers for letters.

How hard are these to cryptanalyze? Caesar? General?

(In)security of substitution ciphers

- ► Key spaces are typically huge. 26 letters ~> 26! possible keys.
- Trivial to crack using frequency analysis (letters, digraphs...)
- Frequencies for English based on data-mining books/articles.


Improvement: Homophonic substitution ciphers

$$\mathcal{A} = \{a, b\}$$

$$H(a) = \{00, 10\}, \text{ and } H(b) = \{01, 11\}.$$

Example

The plaintext ab encrypts to one of 0001, 0011, 1001, 1011.

Improvement: Homophonic substitution ciphers

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- ► Rational: makes frequency analysis more difficult.
- ► Cost: data expansion and more work for decryption.

Polyalphabetic substitution (Leon Alberti, Vignere)



Example: English (n = 26), with k = 3,7,10

 $\mathsf{m}=\mathsf{THI}\;\mathsf{SCI}\;\mathsf{PHE}\;\mathsf{RIS}\;\mathsf{CER}\;\mathsf{TAI}\;\mathsf{NLY}\;\mathsf{NOT}\;\mathsf{SEC}\;\mathsf{URE}$

then

 $E_e(m) =$ WOS VJS SOO UPC FLB WHS QSI QVD VLM XYO

Example: transposition ciphers

C = Aduaenttlydhatoiekounletmtoihahvsekeeeleeyqonouv

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C = Aduaenttlydhatoiekounletmtoihahvsekeeeleeyqonouv

А	n	d	i.	n	t	h	е	е	n
d	t	h	е	Ι	0	V	е	у	0
u	t	а	k	е	i	S	е	q	u
а	Ι	t	0	t	h	е	Ι	0	v
е	у	0	u	m	а	k	е		

Table defines a permutation on 1, ..., 50.

Example: transposition ciphers

C = Aduaenttlydhatoiekounletmtoihahvsekeeeleeyqonouv

А	n	d	i.	n	t	h	е	е	n
d	t	h	е	Ι	0	V	е	у	0
u	t	а	k	е	i.	S	е	q	u
а	Ι	t	0	t	h	е	Ι	0	V
е	у	0	u	m	а	k	е		

Table defines a permutation on 1, ..., 50.

Idea goes back to Greek Scytale: wrap belt spirally around baton and write plaintext lengthwise on it.



Composite ciphers

- Ciphers based on just substitutions or transpositions are not secure
- ► Ciphers can be combined. However ...
 - two substitutions are really only one more complex substitution,
 - ▶ two transpositions are really only one transposition,
 - but a substitution followed by a transposition makes a new harder cipher.
- Product ciphers chain substitution-transposition combinations.
- Difficult to do by hand
 invention of cipher machines.







- Unconditional (information theoretic) security, if key isn't reused!
- Problem?



- Unconditional (information theoretic) security, if key isn't reused!
- ► Problem? Securely exchanging and synchronizing long keys. ^{23 / 63}

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Secret Sharing

Conclusion

ECB vs Others



ECB vs Others



ECB vs Others



One-Wayness (OW)

Put your message in a translucid bag, but you cannot read the text.



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Without the private key, it is computationally **impossible to** recover the plain-text.

One Way Function

- ► Applying *f* is easy
- Computing f^{-1} is difficult



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Factorization

- $p, q \mapsto n = p.q$ easy (quadratic)
- ▶ $n = p.q \mapsto p, q$ difficult

Known Results (1/2)

Fermat's Little Theorem

If a is not divisible by a prime p then p divides $a^{p-1} - 1$

Euclid Theorem

If a prime p divides the product bc then p divides either b or c.

Known Results (2/2)

Chinese Remainder Theorem

Suppose n_1, n_2, \ldots, n_k are positive integers which are pairwise coprime. Then, for any given set of integers a_1, a_2, \ldots, a_k , there exists an integer x solving the system of simultaneous congruences

$$x \equiv a_1 \pmod{n_1} \tag{1}$$

$$x \equiv a_2 \pmod{n_2} \tag{2}$$

$$x \equiv a_k \pmod{n_k} \tag{4}$$

Furthermore, all solutions x to this system are congruent modulo the product $N = n_1 n_2 \dots n_k$.

Hence $x \equiv y \pmod{n_i}$ for all $1 \le i \le k$, if and only if $x \equiv y \pmod{N}$.

RSA 1977 (Rivest, Shamir & Adelman)

Public key: e, nSecret key: p, qwhere n = pq, p and q primes.

- Encryption $c = m^e \mod n$ easy
- ► Decryption $m = c^d \mod n$ difficult where $d = e^{-1} \mod \varphi(n) = (p-1)(q-1)$



RSA, Key generation

- Let p and q two primes
- Let n = pq
- Compute Euler function $\varphi(n) = (p-1)(q-1)$
- Select *e* prime with $\varphi(n)$

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RSA parameters

- ▶ Public key = (n, e)
- Private key = (n, d)

RSA, Decryption (1/3)

If $d = e^{-1} \mod \varphi(n) = (p-1)(q-1)$, why $m = c^d \mod n$

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by definition $ed \equiv 1 \pmod{\varphi(n)}$ hence

RSA, Decryption (1/3)If $d = e^{-1} \mod \varphi(n) = (p-1)(q-1)$, why $m = c^d \mod n$ $C^d \pmod{n} \equiv (M^e)^d \pmod{n} \equiv M^{ed} \pmod{n}$ by definition $ed \equiv 1 \pmod{\varphi(n)}$ hence $ed = 1 + k\varphi(n) = 1 + k(p-1)(q-1), k \in \mathbb{N}$ We have to show using Fermat's Little Theorem: $\forall M \in \mathbb{N}$, $M^{1+k(p-1)(q-1)} \equiv M \pmod{p}$ $M^{1+k(p-1)(q-1)} \equiv M \pmod{q}$

RSA, Decryption (2/3)

• If M is prime with p then, using Fermat's Little Theorem,

 $M^{p-1} \equiv 1 \pmod{p}$ $M^{k(p-1)(q-1)} \equiv 1 \pmod{p}$ $M^{1+k(p-1)(q-1)} \equiv M \pmod{p}$

RSA, Decryption (2/3)

▶ If *M* is prime with *p* then, using Fermat's Little Theorem,

 $M^{p-1} \equiv 1 \pmod{p}$ $M^{k(p-1)(q-1)} \equiv 1 \pmod{p}$ $M^{1+k(p-1)(q-1)} \equiv M \pmod{p}$

• Otherwise p divides M, but p is prime, it means that $I.p \equiv M \equiv 0 \equiv M^{1+k(p-1)(q-1)} \pmod{p}$

Same for q.

RSA, Decryption (1/3)

$$orall M \in \mathbb{N},$$
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RSA, Decryption (1/3)

$$orall M \in \mathbb{N},$$
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 $M^{1+k(p-1)(q-1)} \equiv M \pmod{q}$

p and *q* divide $M^{1+k(p-1)(q-1)} - M$. Moreover *p* and *q* are distinct primes, using Chinese Remainder Theorem, we conclude n = pq divides $M^{1+k(p-1)(q-1)} - M$

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p and q divide $M^{1+k(p-1)(q-1)} - M$. Moreover p and q are distinct primes, using Chinese Remainder Theorem, we conclude n = pq divides $M^{1+k(p-1)(q-1)} - M$

$$C^d \equiv M^{ed} \equiv M^{1+k(p-1)(q-1)} \equiv M \pmod{n}$$

RSA : Rivest, Shamir & Adelman



RSA : Rivest, Shamir & Adelman





Is it secure ?


Is it secure ?







Is it secure ?







 you cannot read the text but you can distinguish which one has been encrypted.

Indistinguishability (IND)

Put your message in a black bag, you can not read anything.



Now a black bag is of course IND and it implies OW.

ElGamal is IND

- $G = (\langle g \rangle, *)$ finite cyclic group of prime order q.
- ► x: private key.
- $y = g^x$: public key.

$$\mathcal{E}(m; r) = (g^r, y^r m) \rightarrow (c, d)$$

 $\mathcal{D}(c, d) = \frac{d}{c^x}$

Is it secure?



Is it secure?





Is it secure?



 It is possible to scramble it in order to produce a new cipher. In more you know the relation between the two plain text because you know the moves you have done.

Non Malleability (NM)

Put your message in a black box.



But in a black box you cannot touch the cube (message), hence NM implies IND.

Summary of Security Notions



Key Privacy or Key Anonymity



Key Privacy or Key Anonymity



Key Privacy or Key Anonymity



SOLUTION

Outline

Motivations

Two Examples

History of Cryptography

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Conclusion

Attacks

Computational Model Cryptanalysis



Attacks

Computational Model Cryptanalysis









Attacks

Computational Model Cryptanalysis



Symbolic Model Logical Attack

Perfect Encryption hypothesis

Needham-Schroeder Public Key Protocol (1978)

"Man in the middle attack" [Lowe'96]



Simple Example



$\{12h10\}_{K_B}$



Simple Example



Simple Example



Simple Example



This kind of attack is valid for all encryptions

Authentication Problem: Wormhole Attack









Example: Needham-Schroeder Protocol 1978





▶ Is *N_B* a shared secret between *A* et *B*?





Lowe Attack on the Needham-Schroeder

so-called "Man in the middle attack"







Agent A

Intruder 1

Agent B

$$\begin{array}{rcccc} A & \longrightarrow & B & : \ \{A, N_a\}_{K_B} \\ B & \longrightarrow & A & : \ \{N_a, N_b\}_{K_A} \\ A & \longrightarrow & B & : \ \{N_b\}_{K_B} \end{array}$$

Lowe Attack on the Needham-Schroeder



Lowe Attack on the Needham-Schroeder



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Needham-Schroeder corrected by Lowe 1995



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Security and Cryptography just by images Logical Attacks

Needham-Schroeder corrected by Lowe 1995



Question

This time the protocol is secure?

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Secret Sharing

Conclusion

Interactive Zero Knowledge Proofs



First, Victor waits outside while Peggy chooses a path.

Interactive Zero Knowledge Proofs



Then Victor enters and shouts the name of a path.

Interactive Zero Knowledge Proofs



At last, Peggy returns along the desired path (using the secret if necessary).

Outline

Motivations

Two Examples

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Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Secret Sharing

Conclusion

Secret Sharing

► How keep nuclear code secret in British Army?



- ► How keep nuclear code secret in British Army?
- Burn it, but do not preseve integrity

How to Share a Secret Code I



1234567



How to Share a Secret Code I



Problem of Integrity and Confidentiality









(2,5)



(3,5)



Security and Cryptography just by images Conclusion

Outline

Motivations

Two Examples

History of Cryptography

Cryptographic Security Intuitions

Logical Attacks

Interactive Zero Knowledge Proofs

Secret Sharing

Conclusion

Security and Cryptography just by images Conclusion

Summary

Today

- Motivation
- History of Cryptography
- Securities notions
- Logical attacks
- Zero knowledge
- Secret Sharing

Security and Cryptography just by images Conclusion

Thank you for your attention



${\sf Questions}\ ?$

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