Towards Automatic Proofs for Symmetric Encryption Modes

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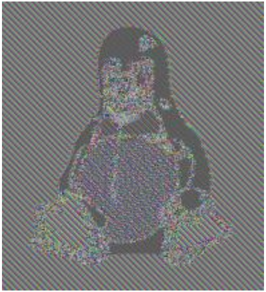
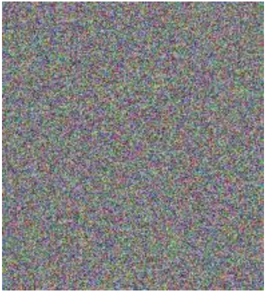
(Work in progress)

Symmetric Encryption of Large Files





Indistinguishability (IND)

The adversary is not able to **guess in polynomial-time even a bit of the plain-text knowing the cipher-text**, notion introduced by S. Goldwasser and S.Micali ([GM84]).

IND for Symmetric Encryption Mode $\mathcal{E}_M$

- Sample $b \xleftarrow{R} \{0,1\}$.
- $(s, m_0, m_1) \xleftarrow{R} \mathcal{A}^{\mathcal{E}_M}(\eta)$
- $b' \xleftarrow{R} \mathcal{A}^{\mathcal{E}_M}(\eta, s, \mathcal{E}_M(m_b))$
- return b'.

Definition

$\mathrm{Adv}_{\mathcal{A}}^{\mathrm{Ind}_{CPA}}(\eta) =$

$$Pr[b' \xleftarrow{R} \mathrm{Ind}_{CPA}^{b=1}(\mathcal{A}) : b' = 1] - Pr[b' \xleftarrow{R} \mathrm{Ind}_{CPA}^{b=0}(\mathcal{A}) : b' = 1]$$

$\mathcal{E}_M$ is IND-CPA secure if $\mathrm{Adv}_{\mathcal{A}}^{ind-CPA}(\eta)$ is negligible for any polynomial-time adversary $\mathcal{A}^{\mathcal{E}_M}$.

Related Works

- Bellare et al, in 1997, propose a Concrete Security Treatment of Symmetric Encryption
- Bellare'04, Shoup'04, Halevi ... have game-based approach
- G. Barthe et al provide formal models of the Generic Model and the ROM in the Coq proof assistant, and prove hardness of the discrete logarithm, security of several schemes.
- R. Corin and J. Den Hartog'06 propose a Hoare-style proof system for game-based cryptographic proofs.
- B. Blanchet develops CryptoVerif security proofs within the game-based, based on observational equivalence.
- A. Datta et al. present a computationally sound compositional logic for key exchange protocols.
- CDELL 08 : Towards Automated Proofs for Asymmetric Encryption Schemes.

Outline

Outline

Block Cipher Modes

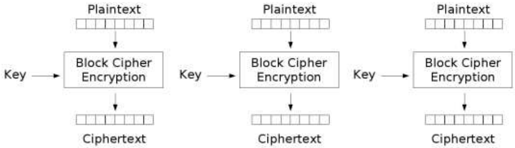
NIST standard

- Electronic Code Book (ECB)
- Cipher Block Chaining (CBC)
- Cipher FeedBack mode (CFB)
- Output FeedBack (OFB), and
- Counter mode (CTR).

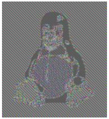
Others

DMC,CBC-MAC, IACBC, IAPM, XCB ,TMAC, HCTR, HCH, EME, EME*, PEP, OMAC, TET, CMC, GCM, EAX, XEX, TAE, TCH, TBC, CCM, ABL4

Each block of the same length is encrypted separately.



Electronic Codebook (ECB) mode encryption



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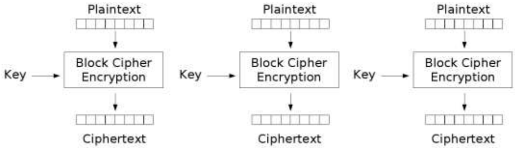
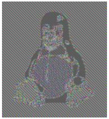
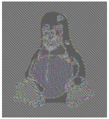
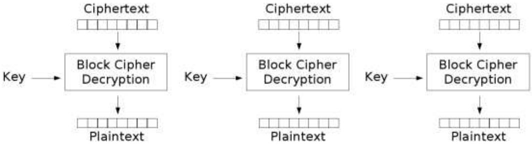


Electronic Codebook (ECB) mode encryption







Electronic Codebook (ECB) mode decryption

Attack on ECB

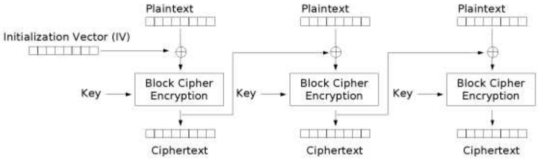
Adversary $A^{\mathcal{E}_K(LR(.,.,b))}$
$M_0 \leftarrow 0^n || 1^n$;
$M_1 \leftarrow 0^{2n}$;
$C[1]C[2] \leftarrow \mathcal{E}_K(LR(M_0, M_1, b))$
If $C[1] = C[2]$ then return 1 else return 0

$$\mathcal{E}_K(LR(m_l, m_r, b)) = \begin{cases} \mathcal{E}_K(m_l) & if\ \ b = 1 \\ \mathcal{E}_K(m_r) & if\ \ b = 0 \end{cases}$$

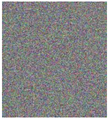
$C[i]$ denotes the i-th block of a string C.

$$Adv_{\mathcal{SE}}^{IND-CPA}(A) = 1 - 0 = 1$$

Cipher Block Chaining (CBC) Encryption

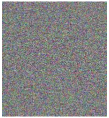


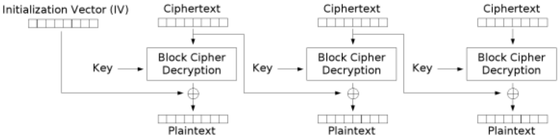
Cipher Block Chaining (CBC) mode encryption



$$C_i = E_K(P_i \oplus C_{i-1}), C_0 = IV$$

Cipher Block Chaining (CBC) Decryption





Cipher Block Chaining (CBC) mode decryption

$$P_i = D_K(C_i) \oplus C_{i-1}, C_0 = IV$$

CBC and others

CBC	CTR	OFB	CFB
$IV \xleftarrow{\$} \mathcal{U};$	$IV \xleftarrow{\$} \mathcal{U};$	$IV \xleftarrow{\$} \mathcal{U};$	$IV \xleftarrow{\$} \mathcal{U};$
$z_1 := IV \oplus m_1;$	$z_1 := \mathcal{E}(IV+1);$	$z_1 := \mathcal{E}(IV);$	$z_1 := \mathcal{E}(IV);$
$c_1 := \mathcal{E}(z_1);$	$c_1 := m_1 \oplus z_1;$	$c_1 := m_1 \oplus z_1;$	$c_1 := m_1 \oplus z_1;$
$z_2 := c_1 \oplus m_2;$	$z_2 := \mathcal{E}(IV+2);$	$z_2 := \mathcal{E}(z_1);$	$z_2 := \mathcal{E}(c_1);$
$c_2 := \mathcal{E}(z_2);$	$c_2 := m_2 \oplus z_2;$	$c_2 := m_2 \oplus z_2;$	$c_2 := m_2 \oplus z_2;$
$z_3 := c_2 \oplus m_3;$	$z_3 := \mathcal{E}(IV+3);$	$z_3 := \mathcal{E}(z_2);$	$z_3 := \mathcal{E}(c_2);$
$c_3 := \mathcal{E}(z_3);$	$c_3 := m_3 \oplus z_3;$	$c_3 := m_3 \oplus z_3;$	$c_3 := m_3 \oplus z_3;$

Outline

Grammar

$$
\begin{aligned}
\mathsf{c} \quad ::= \quad & x \xleftarrow{\$} \mathcal{U} \mid x := \mathcal{E}(y) \mid x := \mathcal{E}^{-1}(y) \\
& \mid x := y \oplus z \mid x := y \| z \mid x := y[n, m] \\
& \mid x := y + 1 \mid c_1 ; c_2
\end{aligned}
$$

Generic Encryption Mode

Definition

A generic encryption mode M is represented by

$$\mathcal{E}_M(m_1|\ldots|m_p, c_0|\ldots|c_p) : \textbf{var } \vec{x}; \mathrm{c}$$

$$\begin{array}{l}
\mathcal{E}_{CBC}(m_1|m_2|m_3, IV|c_1|c_2|c_3) : \\
\textbf{var } z_1, z_2, z_3; \\
IV \xleftarrow{\$} \mathcal{U}; \\
z_1 := IV \oplus m_1; \\
c_1 := \mathcal{E}(z_1); \\
z_2 := c_1 \oplus m_2; \\
c_2 := \mathcal{E}(z_2); \\
z_3 := c_2 \oplus m_3; \\
c_3 := \mathcal{E}(z_3);
\end{array}$$

Outline

Predicates

$$\psi ::= \mathrm{Indis}(\nu x; V) \mid F(e) \mid \mathrm{E}(\mathcal{E}, e) \mid Rcounter(e)$$
$$\varphi ::= \mathrm{true} \mid \varphi \wedge \varphi \mid \psi,$$

Indis$(\nu x; V)$: any adversary has negligible probability to distinguish whether he is given results of computations performed using the value of x or a random value, when he is given the values of the variables in V.

$F(e)$: means e is a fresh random value.

$\mathrm{E}(\mathcal{E}, l)$: the probability that the values of the expression e_i in the list l have been submitted to the symmetric encryption $\mathcal{E}$ is negligible.

$RCounter(e)$: means that e is the most recent value of a counter that started at a fresh random value.

Remark:

$$F(e) \Rightarrow \mathrm{Indis}(\nu e)$$

$$F(e) \Rightarrow \mathrm{E}(\mathcal{E}, e)$$

More Formally

- $X \models \text{true}$.
- $X \models \varphi \wedge \varphi'$ iff $X \models \varphi$ and $X \models \varphi'$.
- $X \models \text{Indis}(\nu x; V)$ iff $[u \stackrel{r}{\leftarrow} \mathcal{U}; (S, \mathcal{E}) \stackrel{r}{\leftarrow} X : (S(u, V), \mathcal{E})] \sim [(S, \mathcal{E}) \stackrel{r}{\leftarrow} X : (S(x, V), \mathcal{E})]$
- $X \models \text{E}(\mathcal{E}, I)$ iff for all $e \in I$, $\Pr[(S, \mathcal{E}) \stackrel{r}{\leftarrow} X : S(e) \in S(\mathcal{T}_{\mathcal{E}}).dom]$ is negligible.
- $X \models F(e)$ iff $e \in S(F)$.
- $X \models RCounter(e)$ iff $e \in S(C)$.

Semantic of the Programming Language

$$[\![x \xleftarrow{r} \mathcal{U}]\!](S,\mathcal{E}) = [u \xleftarrow{r} \mathcal{U} : (S\{x \mapsto u, F \mapsto F \cup \{x\}\}, \mathcal{E})]$$

$$[\![x := \mathcal{E}(y)]\!](S,\mathcal{E}) = \begin{cases} \delta(S\{x \mapsto v, F \mapsto F \cup \{x\} \setminus \{y\}\}, \mathcal{E}) \text{ if } (S(y), v) \in \mathcal{T}_{\mathcal{E}} \\ \delta(S\{x \mapsto v, F \mapsto F \cup \{x\} \setminus \{y\}, \mathcal{T}_{\mathcal{E}} \mapsto S(\mathcal{T}_{\mathcal{E}}) \cdot (S(y), v)\}, \mathcal{E}) \text{ if } (S(y), v) \notin \mathcal{T}_{\mathcal{E}} \\ \qquad \text{and } v = \mathcal{E}(S(y)) \end{cases}$$

$$[\![x := \mathcal{E}^{-1}(y)]\!](S,\mathcal{E}) = \delta(S\{x \mapsto \mathcal{E}^{-1}(S(y)), F \mapsto F \setminus \{x, y\}\}, \mathcal{E})$$

$$[\![x := y \oplus z]\!](S,\mathcal{E}) = \delta(S\{x \mapsto S(y) \oplus S(z), F \mapsto F \setminus \{x, y, z\}\}, \mathcal{E})$$

$$[\![x := y||z]\!](S,\mathcal{E}) = \delta(S\{x \mapsto S(y)||S(z), F \mapsto F \setminus \{x, y, z\}\}, \mathcal{E})$$

$$[\![x := y[n,m]]\!](S,\mathcal{E}) = \delta(S\{x \mapsto S(y)[n,m], F \mapsto F \setminus \{x, y\}\}, \mathcal{E})$$

$$[\![x := y + 1]\!](S,\mathcal{E}) = \begin{cases} \delta(S\{x \mapsto S(y)+1, C \mapsto C \cup \{x\} \setminus \{y\}, F \mapsto F \setminus \{x, y\}\}, \mathcal{E}) \text{ if } y \in F \text{ or } y \in C \\ \delta(S\{x \mapsto S(y)+1, F \mapsto F \setminus \{x, y\}\}, \mathcal{E}) \text{ otherwise} \end{cases}$$

$$[\![c_1; c_2]\!] = [\![c_2]\!] \circ [\![c_1]\!]$$

Main Result

Prop

*Let $\mathcal{E}_M(m_1|\ldots|m_p, c_0|\ldots|c_p)$: **var** $\vec{x}; c$ be a generic encryption mode, and let $IO = \{m_1, \ldots, m_p, c_0, \ldots, c_p\}$. Then $\mathcal{E}_M$ is IND-CPA secure, if $\{true\}c \bigwedge_{i=0}^{i=p}\{Indis(\nu c_i; IO)\}$ is valid.*

Proof in progress.

Outline

Random Assignment:

- (R1) $\{true\}\ x \xleftarrow{\$} \mathcal{U}\ \{F(x)\}$
- (R2) $\{\mathrm{Indis}(\nu y; V)\}\ x \xleftarrow{\$} \mathcal{U}\ \{\mathrm{Indis}(\nu y; V, x)\}$

Using Lemma and (R1) we obtain $\{\mathrm{Indis}(\nu x)\}$ and $\{\mathrm{E}(\mathcal{E}, x)\}$, this combination is often used in the examples.

Block Cipher Rules:

- (B1) $\{\mathsf{E}(\mathcal{E}, y)\}\ x := \mathcal{E}(y)\ \{F(x)\}$
- (B2) $\{\mathsf{Indis}(\nu y; V)\}\ x := \mathcal{E}(y)\ \{\mathsf{Indis}(\nu y; V)\}$
- (B3) $\{Rcounter(y)\}\ x := \mathcal{E}(y)\ \{Rcounter(y)\}$

Increment:

- (I1) $\{F(y)\}\ x := y + 1$
  $\{RCounter(x)\} \wedge \{\mathrm{E}(\mathcal{E}, x)\} \wedge \{\mathrm{Indis}(\nu y; \mathrm{Var} - x)\}$
- (I2) $\{RCounter(y)\}\ x := y + 1\ \{RCounter(x)\} \wedge \{\mathrm{E}(\mathcal{E}, x)\}$
- (I3) $\{\mathrm{Indis}(\nu z; V)\}\ x := y + 1\ \{\mathrm{Indis}(\nu z; V - x)\}$ if $z \neq x, y$ and $y \notin V$

Xor operator:

- (X1) $\{\text{Indis}(\nu y; V, y, z)\} x := y \oplus z \{\text{Indis}(\nu x; V, x, z)\}$,
- (X2) $\{\text{Indis}(\nu y; V, x)\} x := y \oplus z \{\text{Indis}(\nu y; V)\}$,
- (X3) $\{\text{Indis}(\nu t; V, y, z)\}\ x := y \oplus z\ \{\text{Indis}(\nu t; V, x, y, z)\}$ if $t \neq x, y, z$
- (X4) $\{F(y)\}\ x := y \oplus z\ \{\text{E}(\mathcal{E}, x, y)\}$ if $y \neq z$

Concatenation:

- (C1) $\{\mathsf{Indis}(\nu y; V, y, z)\} \wedge \{\mathsf{Indis}(\nu z; V, y, z)\}\ x := y \| z$ $\{\mathsf{Indis}(\nu x; V, x)\}$ if $y, z \notin V$
- (C2) $\{\mathsf{Indis}(\nu t; V, y, z)\}\ x := y \| z\ \{\mathsf{Indis}(\nu t; V, x, y, z)\}$ if $t \neq x, y, z$

Generic preservation rules:

The following rules are sound, when $x \notin V$. Assume that $z \neq x, w, v$ and c is either $x \xleftarrow{\$} \mathcal{U}$, $x := w \| v$, $x := w \oplus v$, $x := \mathcal{E}(w)$ or $x := w + 1$:

- (G1) $\{\mathrm{Indis}(\nu z; V)\}$ c $\{\mathrm{Indis}(\nu z; V)\}$ provided c is not $x := w + 1$
- (G2) $\{\mathrm{E}(\mathcal{E}, z)\}$ c $\{\mathrm{E}(\mathcal{E}, z)\}$
- (G3) $\{RCounter(z)\}$ c $\{RCounter(z)\}$
- (G4) $\{F(z)\}$ c $\{F(z)\}$

Outline

Example: CBC

$\mathcal{E}_{CBC}(m_1|m_2|m_3, IV|c_1|c_2|c_3)$

var IV, z_1, z_2, z_3;

$IV \xleftarrow{\$} \mathcal{U}$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var})\} \wedge F(IV) \wedge \{\mathrm{E}(\mathcal{E}, IV)\}$	(R1)
$z_1 := IV \oplus m_1$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var} - z_1)\} \wedge \{\mathrm{E}(\mathcal{E}, z_1, IV)\}$	(X2)(X4)
$c_1 := \mathcal{E}(z_1)$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var} - z_1)\}$	(G1)
	$\wedge\{\mathrm{Indis}(\nu c_1; \mathrm{Var})\} \wedge \{F(c_1)\}$	(B1)
$z_2 := c_1 \oplus m_2$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var} - z_1)\}$	(G1)
	$\wedge\{\mathrm{Indis}(\nu c_1; \mathrm{Var} - z_2)\} \wedge \{\mathrm{E}(\mathcal{E}, z_1, c_1)\}$	(X2)(X4)
$c_2 := \mathcal{E}(z_2)$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var} - z_1)\} \wedge \{\mathrm{Indis}(\nu c_1; \mathrm{Var} - z_2)\}$	(G1)
	$\wedge\{Indis(\nu c_2; \mathrm{Var})\} \wedge F(c_2)$	(B1)
$z_3 := c_2 \oplus m_3$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var} - z_1)\} \wedge \{\mathrm{Indis}(\nu c_1; \mathrm{Var} - z_2)\}$	(G1)
	$\wedge\{\mathrm{Indis}(\nu c_2; \mathrm{Var} - z_3)\} \wedge \{\mathrm{E}(\mathcal{E}, z_3, c_3)\}$	(X2)(X4)
$c_3 := \mathcal{E}(z_3)$;	$\{\mathrm{Indis}(\nu IV; \mathrm{Var} - z_1)\} \wedge \{\mathrm{Indis}(\nu c_1; \mathrm{Var} - z_2)\}$	(G1)
	$\wedge\{\mathrm{Indis}(\nu c_2; \mathrm{Var} - z_3)\} \wedge \{Indis(\nu c_3; \mathrm{Var})\}$	(B1)

Outline

Summary

- Generic Encryption Mode
- New predicats
- Hoare Logic for proving generic encryption mode IND-CPA
- Automatic proof of CBC, FBC, OFB CFB.

Future Works

- Hybrid encryption
- using LSFR (Dual Encryption Mode)
- using Hash function
- using mathematics (GMC)
- IND-CCA ?
  Desai 2000: New Paradigms for Constructing Symmetric Encryption Schemes Secure against Chosen-Ciphertext Attack

Thank you for your attention.

Questions ?