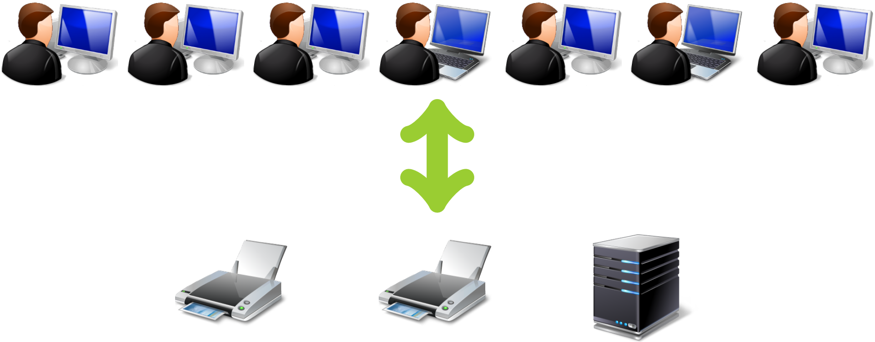
Concurrency in Snap-Stabilizing Local Resource Allocation

Karine Altisen Stéphane Devismes **Anaïs Durand**

May ??, 2015

Resource Allocation Problems

n processes, k resources, $n \gg k$



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- Code to access a resource
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With Several Resources ($k > 1$)

- **Concurrency:** Maximize the utilization of the resources

ℓ-Exclusion [Fischer *et al*, 79]

ℓ **identical** copies of a non-shareable reusable resource

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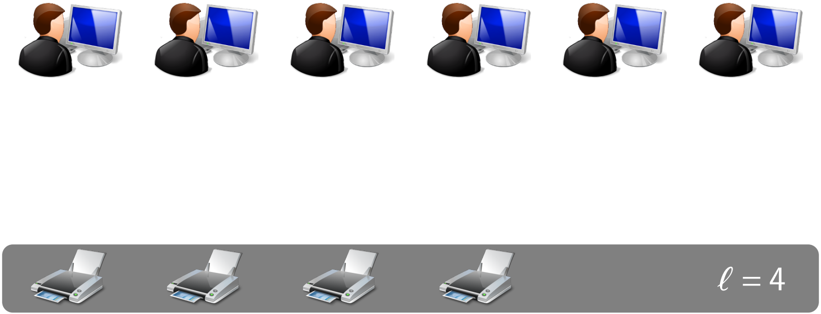
Properties

- **Safety:** $|\{\text{processes concurrently in CS}\}| \leq \ell$
- **Fairness:** Requesting process eventually enters CS
- **Avoiding ℓ-Deadlock:**
  If $|\{\text{processes concurrently in CS}\}| < \ell$

  then
  - a requesting process can obtain CS
  - **even if no process leaves CS meanwhile**

Avoiding ℓ-Deadlock [Fischer *et al*, 79]



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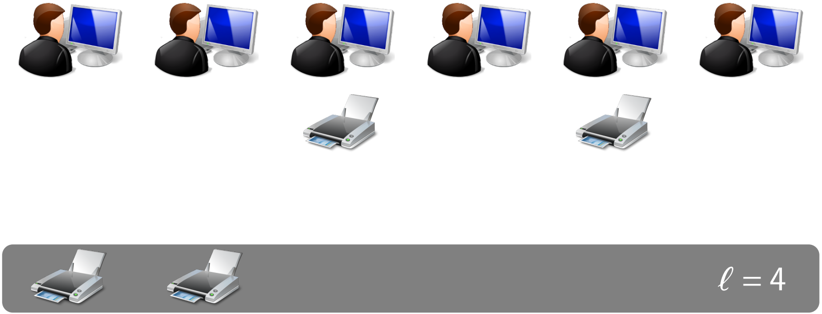

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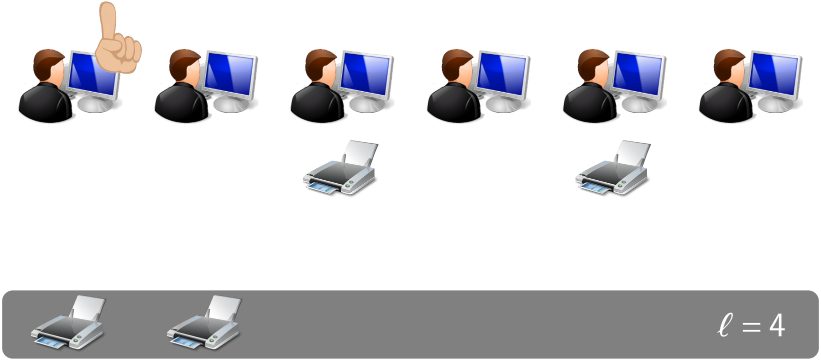

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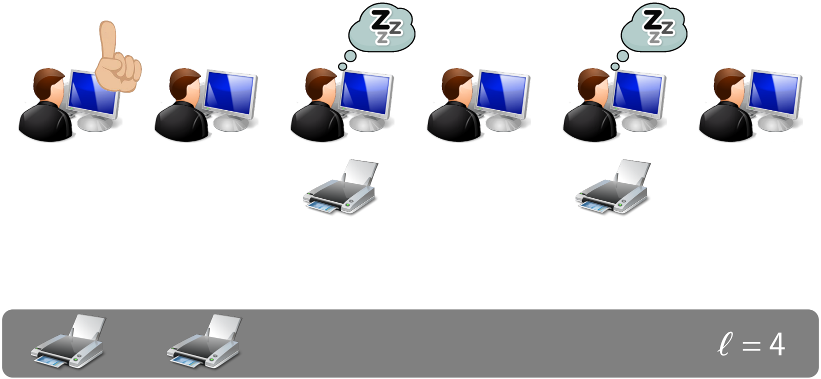

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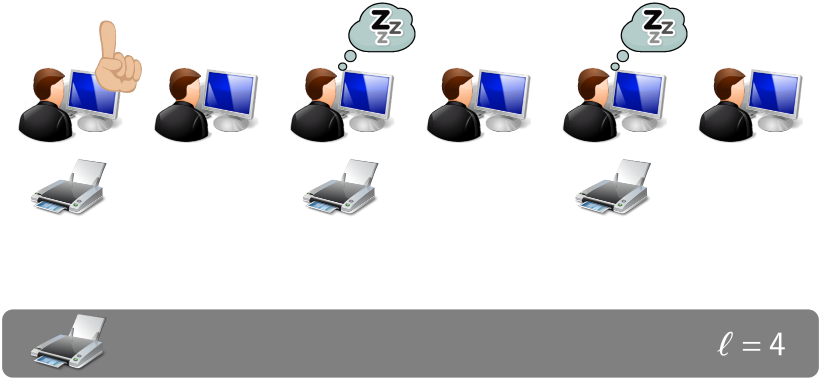

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- But, often **not considered** in correctness proofs of resource allocation algorithms.

Several Properties Handling Concurrency

- **Avoiding ℓ-Deadlock:**

  ℓ-exclusion problem [Fischer *et al*, 79]

- **(k,ℓ)-Liveness:**

  k-out-of-ℓ-exclusion problem [Datta *et al*, 03]

- **Maximal-Concurrency:**

  Committee coordination problem [Bonakdarpour *et al*, 11]

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Drawback : **dedicated** to a specific problem

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Generalization of the previous properties

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Equivalent Definition of Maximal-Concurrency

If CSs last a **long enough time**

then eventually $P_{FREE} = \emptyset$

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Local Resource Allocation (LRA) [Cantarell *et al*, 03]

Generalization of Many Classical Problems

- Dining Philosophers
- Local Mutual Exclusion
- Drinking Philosophers
- Local Reader/Writer
- Local Group Mutual Exclusion
- ...

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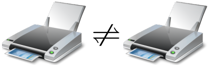

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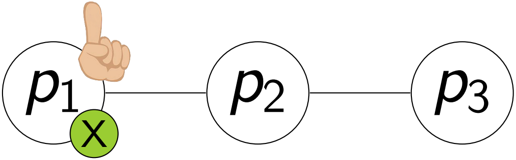
🖨 $\neq$ 🖨

Example: Local Reader-Writer Problem

👁 $\rightleftharpoons$ 👁   👁 $\neq$ ✏   ✏ $\neq$ ✏

Impossibility Result: Maximal-Concurrency in LRA

Two resources: X $\neq$ Y



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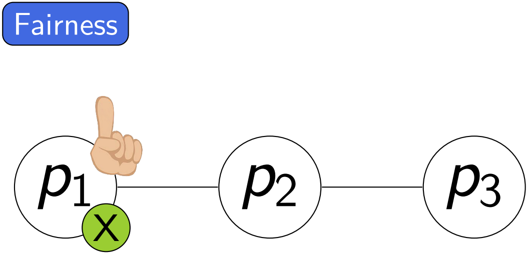

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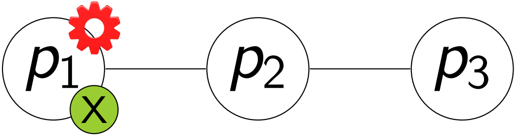

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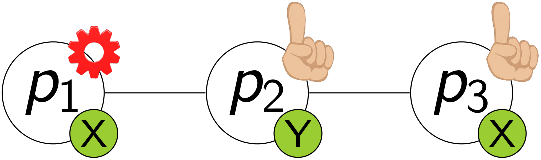

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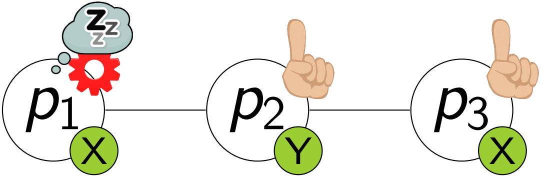

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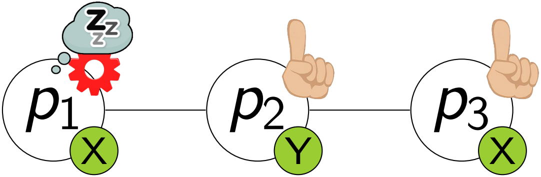

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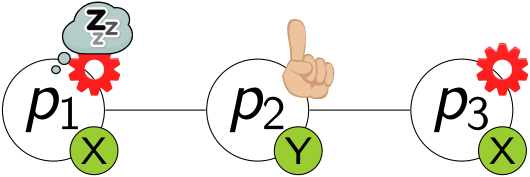

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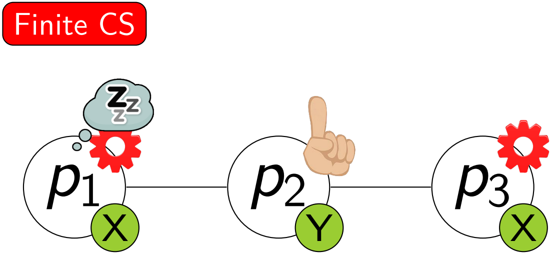
Two resources: $\mathsf{X} \not\rightleftharpoons \mathsf{Y}$



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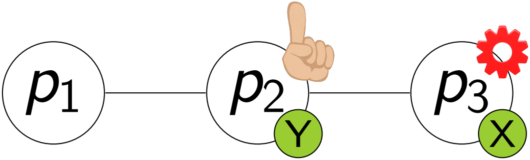

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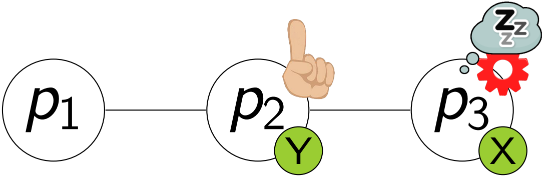

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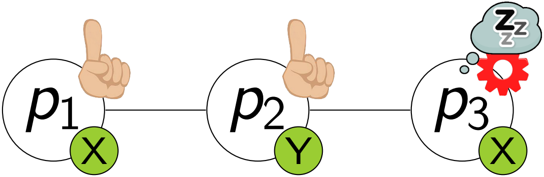

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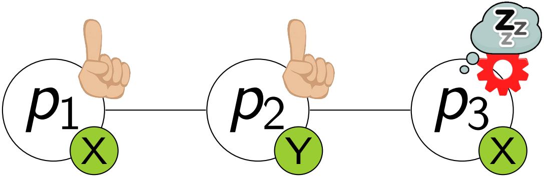

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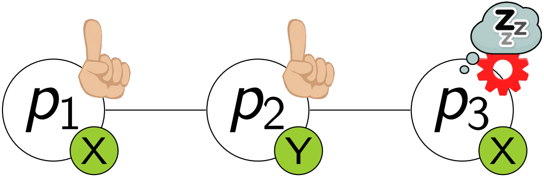

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Fairness property violated

(Strong) Partial Maximal-Concurrency

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Partial Maximal-Concurrency, Parameter: X

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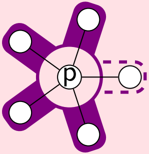
Strong Partial Maximal-Concurrency

Partial Maximal-Concurrency with
$X =$ neighbors of **a unique** process **minus one**.



Snap-Stabilizing LRA Algorithm

Requirements

Locally Shared Memory Model

- Locally shared variables
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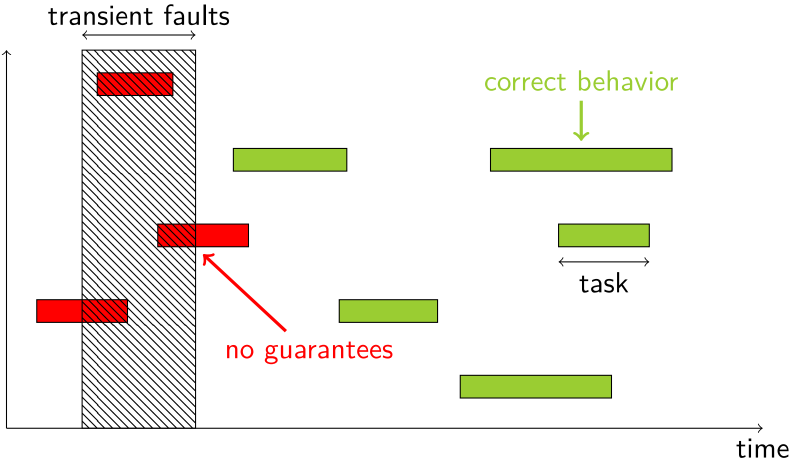
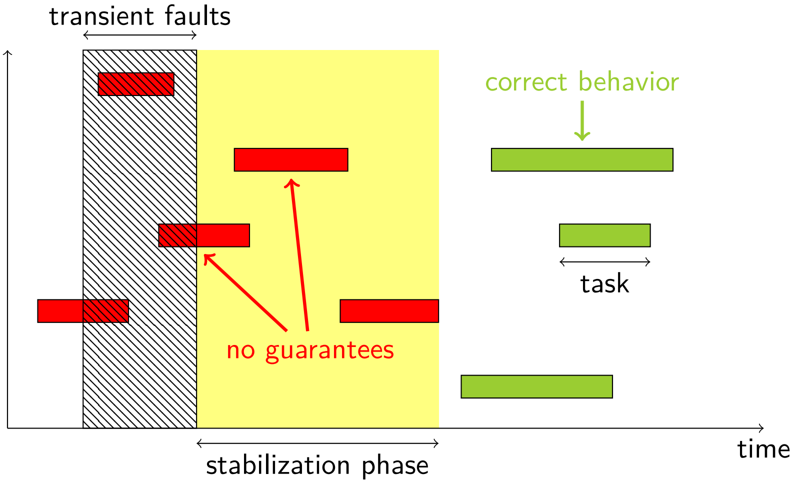
Network

- Connected
- Bidirectional
- Identified

Snap-Stabilization [Bui *et al*, 07]



Self-Stabilization [Dijkstra, 74]



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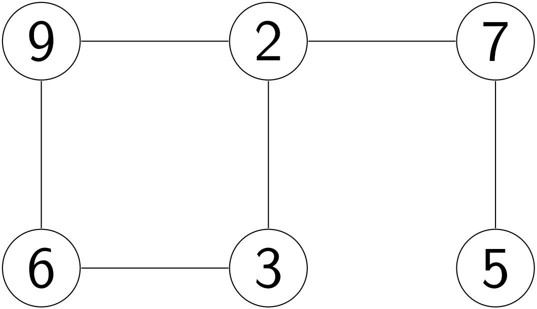
Snap-Stabilizing LRA Algorithm

Example on the Local Reader-Writer Problem



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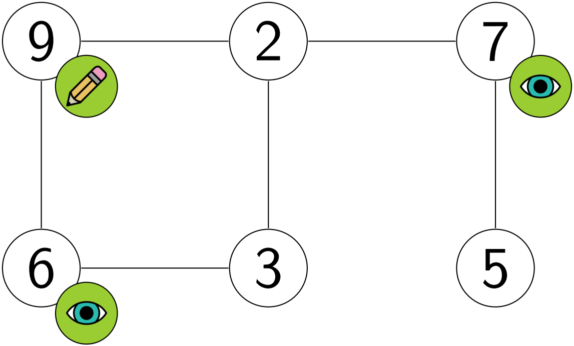

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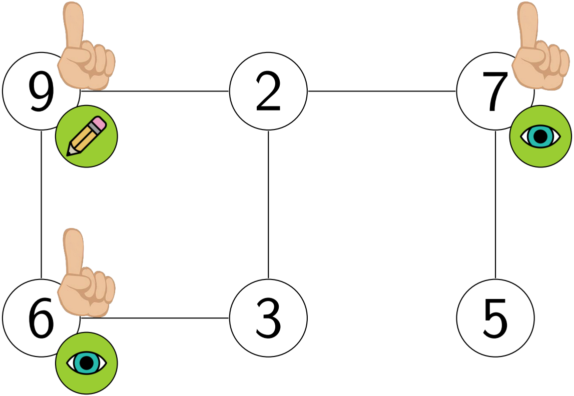

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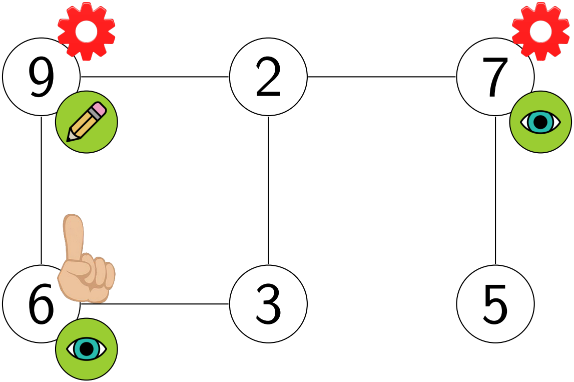

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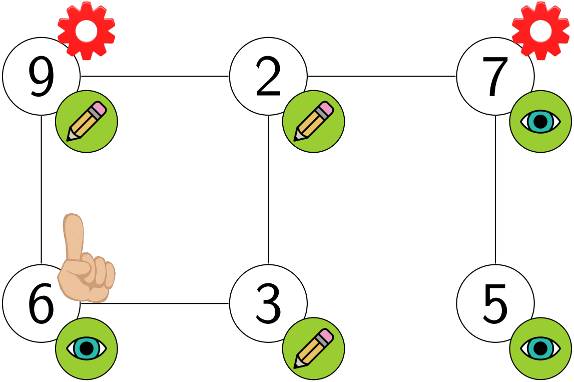

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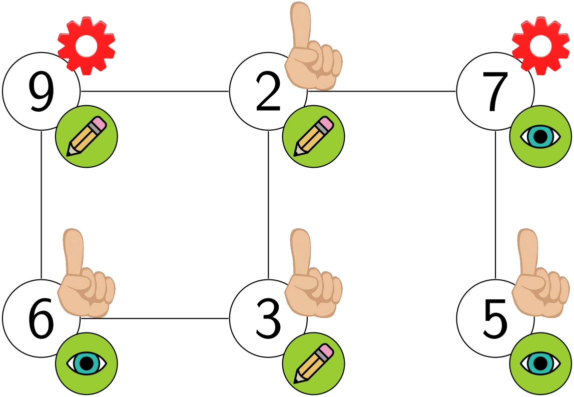

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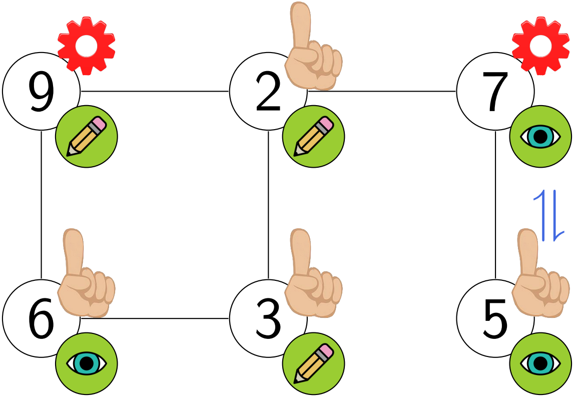

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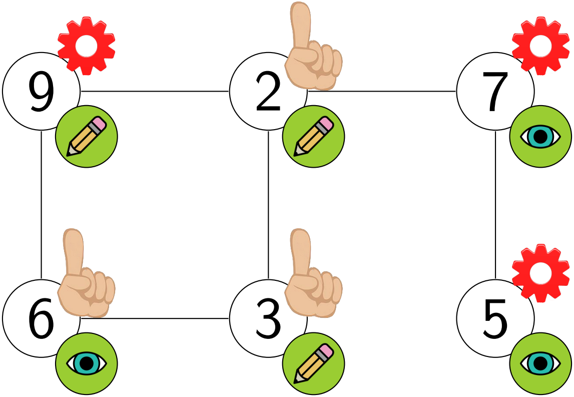

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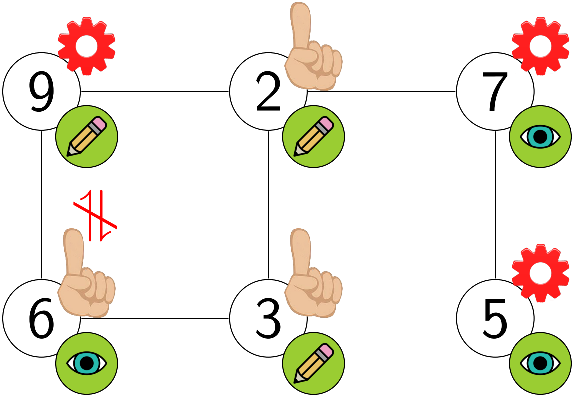

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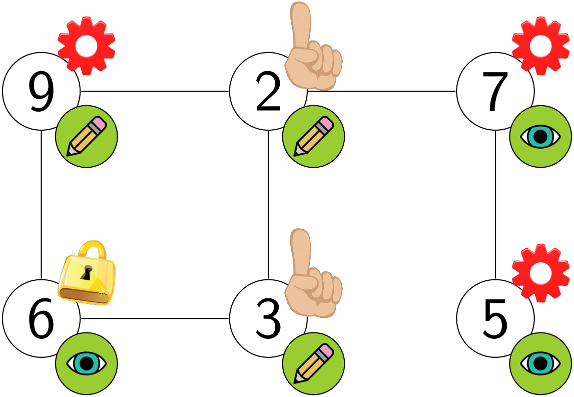

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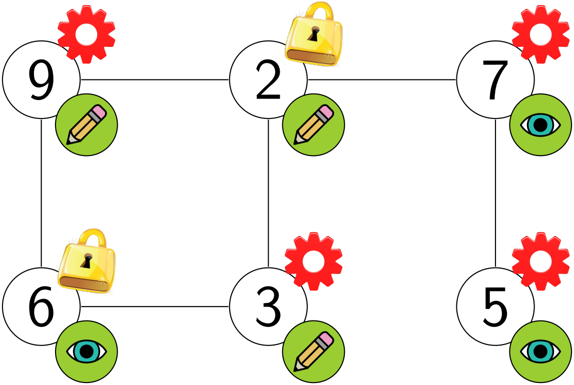

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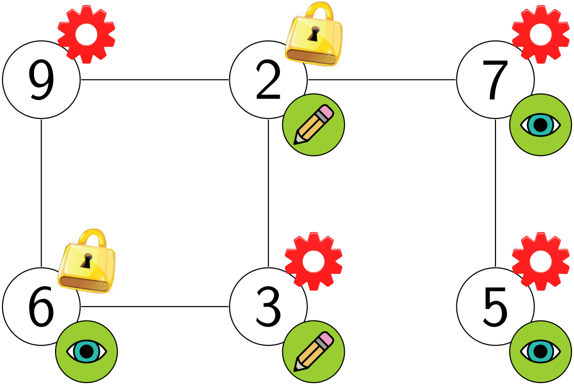

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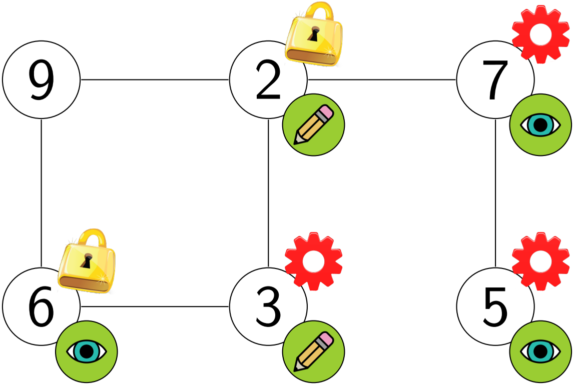

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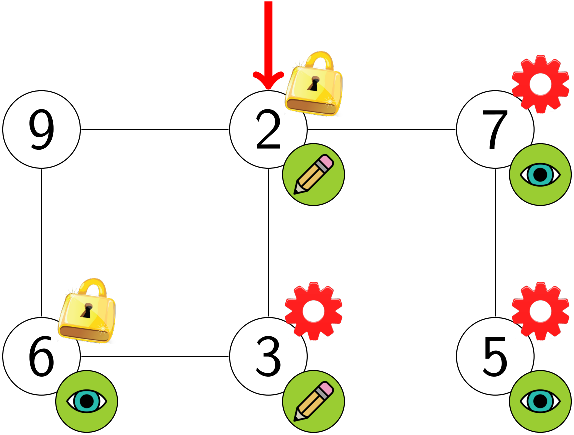
Example on the Local Reader-Writer Problem

Local minimum ⇒ may never enter its CS



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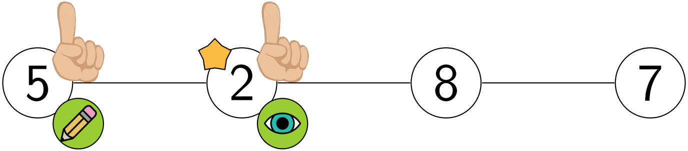

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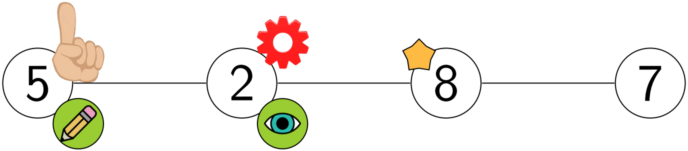

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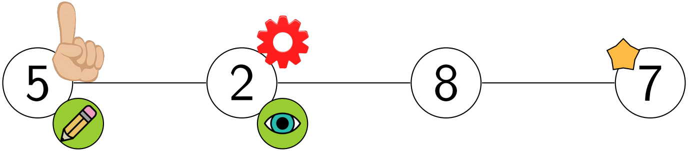

Snap-Stabilizing LRA Algorithm: Fairness

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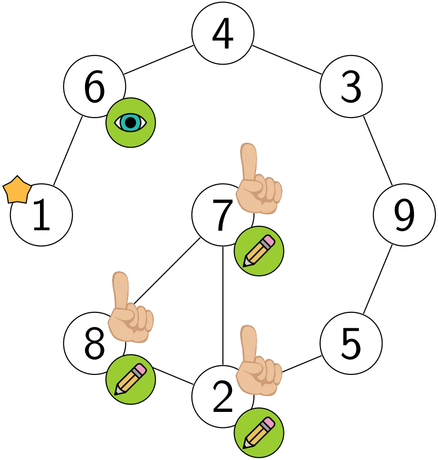

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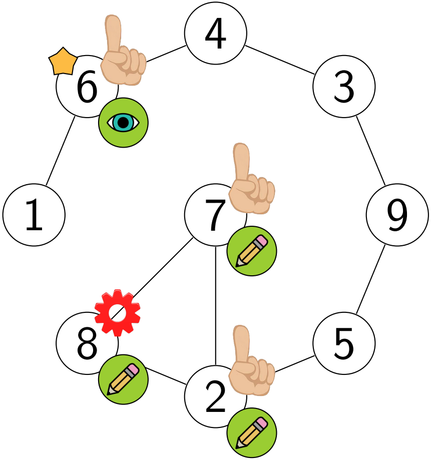

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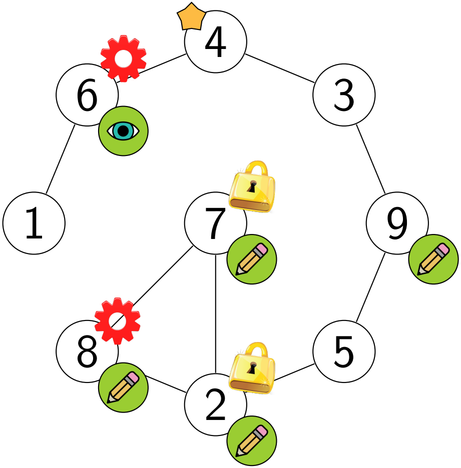

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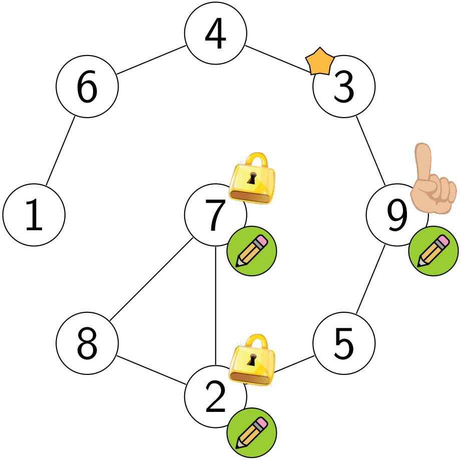

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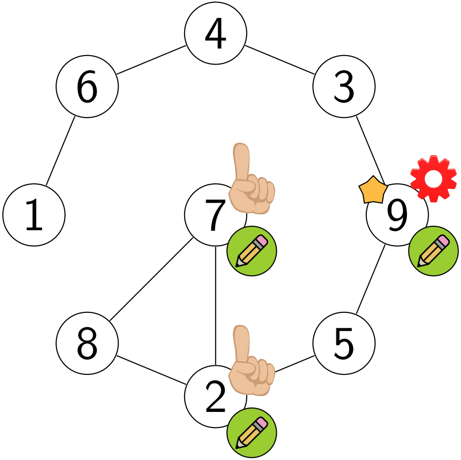

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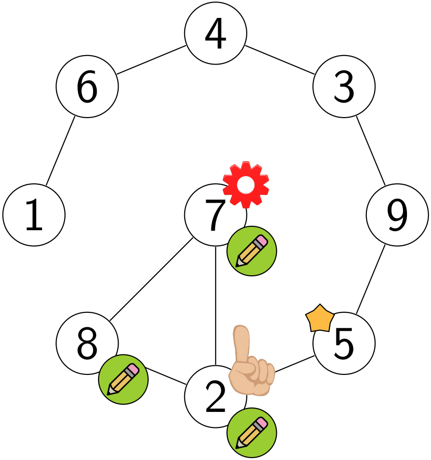

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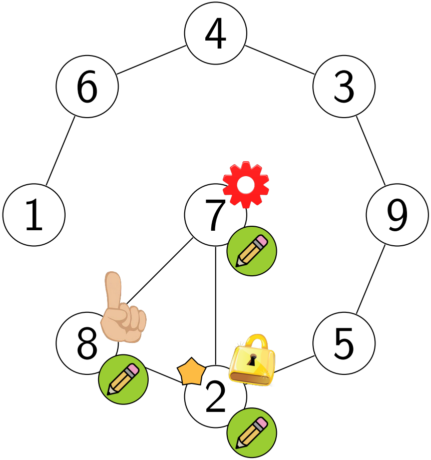

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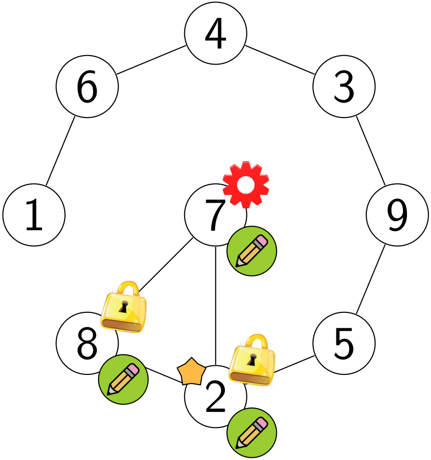

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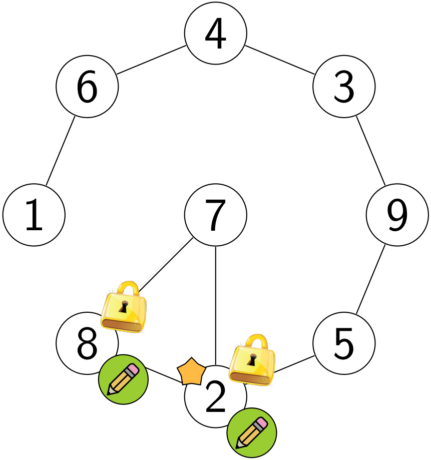

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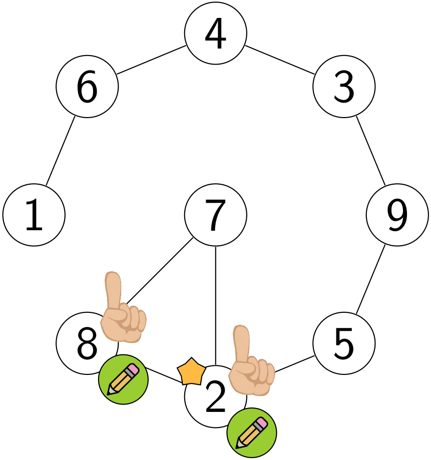

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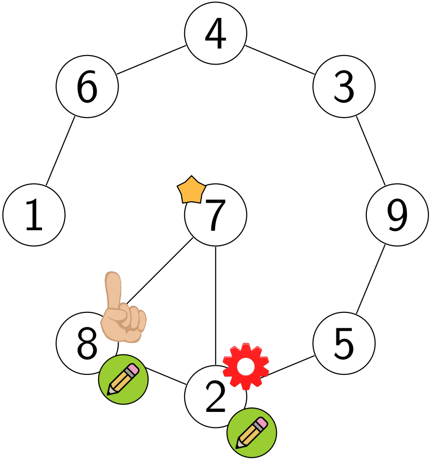
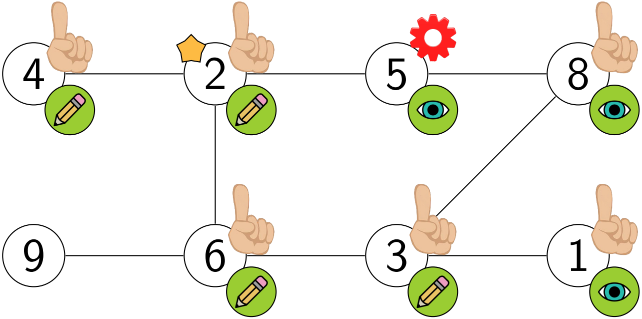


Snap-Stabilizing LRA Algorithm: Strong Partial Maximal-Concurrency

A **worst case** in the Local Reader-Writer Problem



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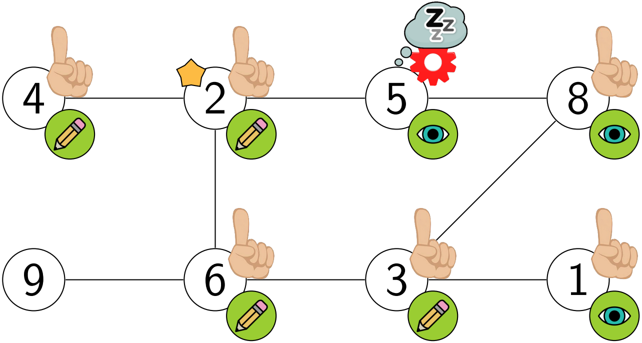

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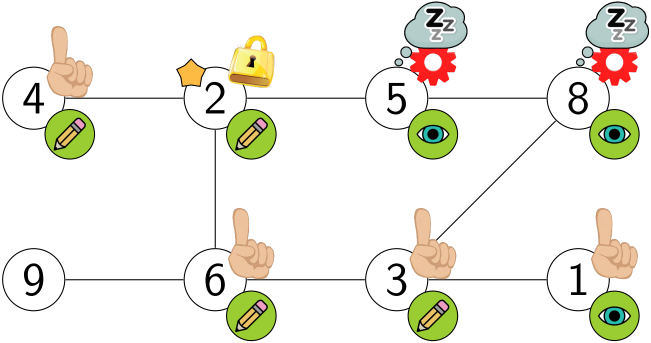

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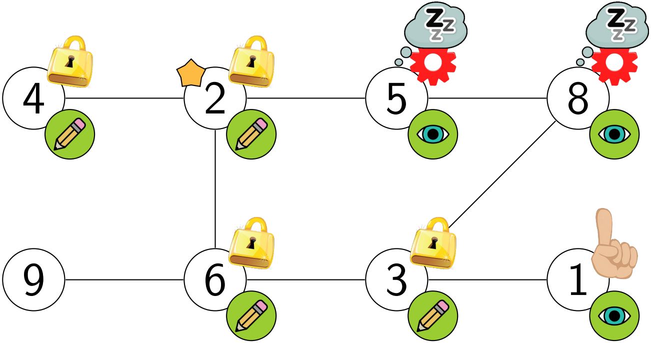

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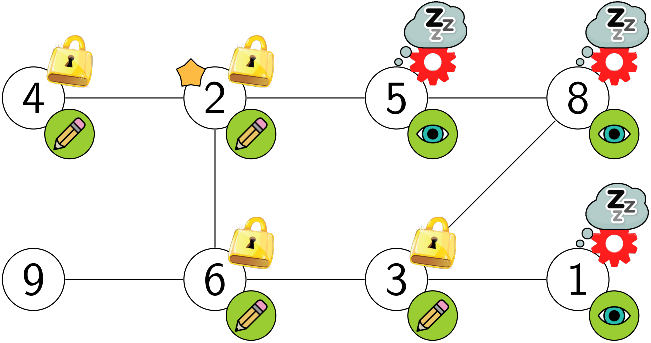

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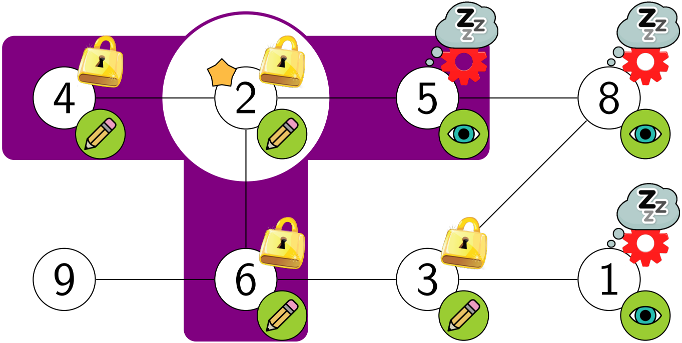

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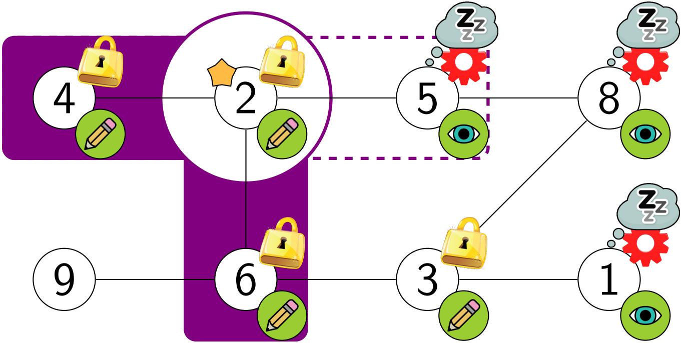

Conclusion

Contributions

- Definition of the **maximal-concurrency**
- Proof of **impossibility** of maximal-concurrency in LRA
- Definition of the **(strong) partial maximal-concurrency**
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Perspectives

Define the class of resource allocation problems where maximal-concurrency/strong partial-maximal concurrency can be achieved.

Thank you for your attention.

Do you have any questions ?